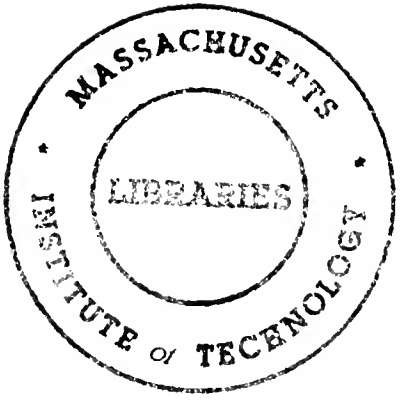
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A THEORY OF CONSUMER PROMOTIONS: THE MODEL

Leigh McAlister
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ABSTRACT

Consumers who react to a promotional offer might switch from their preferred brand to the promoted brand, they might stockpile the promoted brand, or they might switch and stockpile. We model the implications of such consumer responses for a manufacturer's profit. Conditions under which a profit maximizing manufacturer should promote its product are then inferred from the aggregated response function. It is never profitable to promote to consumers who only stockpile in response to a promotion. A sufficient number of consumers must switch to the promoted brand to make promoting profitable. Stockpiling behavior on the part of those consumers who switch to the promoted brand will, however, enhance profitability. The paradigm provides further insights which run counter to prevailing theory. Promotion is shown to be more likely to benefit small, rather than large, share brands. And we conjecture that there should not be a pronounced trough in market level sales following a promotional sales peak.

Sales promotion expenditures have grown dramatically in recent years. In 1969, promotional spending approximately equalled advertising spending. Since then, total promotional spending has grown to $60 billion and the promotion/advertising ratio has risen to 60/40 (See Figure 1).

[FIGURE 1 ABOUT HERE]

Some industry sources find this trend alarming. Among them is Hertz Chairman Frank Olson, who claims that promotional gifts associated with car rentals "do nothing but increase the cost of doing business".[1] Philip K. Fricke, a Goldman, Sachs and Co. auto stocks analyst, sees a similar futility in new car rebates. Fricke proposes that "the auto industry's on-again, off-again pattern of rebates [has not stimulated the desired industry upturn, instead it]...has conditioned the consumer to wait -- almost indefinitely -- for some giveaway."[2]

Robert G. Brown, president of SPAR, holds a more moderate opinion. He suggests that promotions can be profitable but warns of the "trade promotions trap." He claims that "a promotion forced [sales] increment this year often undermines next year's performance and even the long term viability of the brand."[3]

Other industry sources see promotional spending as fundamentally sound. William E. Winter, Chairman of the Board for 7-Up, told the Association of National Advertisers in 1980 that promotion should no longer be "seen as a short-term benefit for the new or ailing product, but as a contributor to the long term marketing growth of established goods and services. When the building blocks for today's marketing programs are assembled", he maintains, "promotion is going into the foundation."[4]

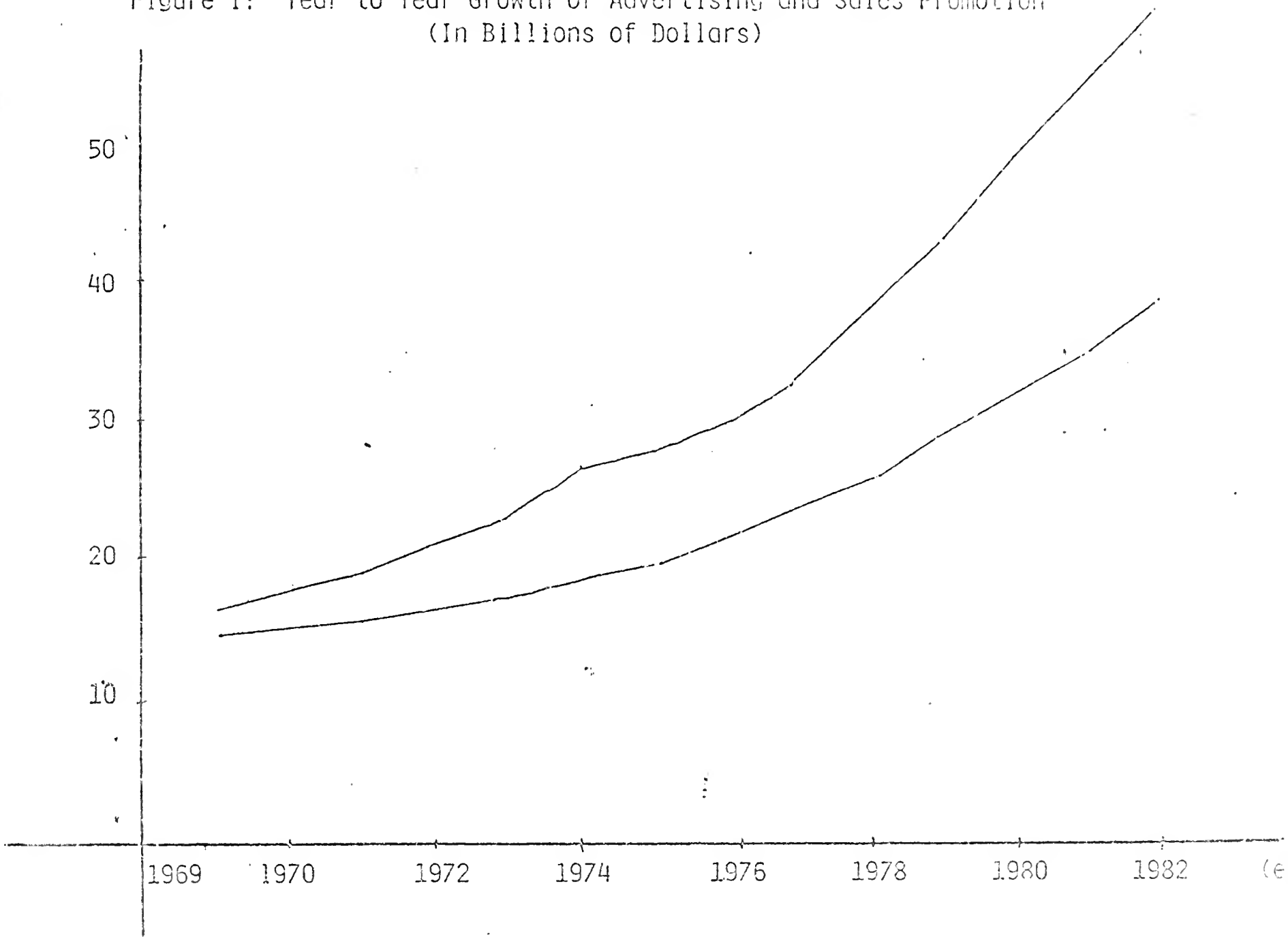
The objective of this paper is to develop a model of consumer promotions that can accomodate these seemingly discrepant views. The model proposed is based on the reactions of consumers to promotional offers. It is used to characterize the conditions under which a profit maximizing manufacturer should engage in promotional activity. Those conditions, a function of the particular consumer mix, may vary across product classes, across products within a class and even a cross promotional instruments for a particular product.

We begin the paper with a review of the literature which considers the efficacy and importance of promotional activity. The ideas drawn from that literature are tied together in an analytical model of market response to manufacturers' promotions. We analytically characterize those product classes in which promotional activity is profitable. We show that promotions are more likely to benefit smaller share brands and that there should be no pronounced trough following a promotional sales peak. In the summary and conclusions we briefly report the results of a related paper based on this model. That work addresses the problems of allocating promotional resources across products within a product line and sequencing promotional activity across time.

LITERATURE REVIEW

Paralleling the increased spending on promotion by industry has been a burgeoning of research on the topic by academics. One thrust in that research has been the investigation of market response to different promotional devices (e.g., price deals, Massy and Frank 1965; premiums, Seipel 1971; in-store display, Chevalier 1975; coupons, Klein 1981). Chevalier and Curhan (1976) examined the manufacturers' ability to stimulate the use of such promotional devices by retailers. Brown (1974) and Sunoo and Lin (1978) considered the interaction effect of advertising and promotion on market response.

Figure 1: Year to Year Growth of Advertising and Sales Promotion
(In Billions of Dollars)



Source: Russel D. Bowman, John C. Young and John Adler, "Improving the Payout of the Advertising Promotion Mix," (New York: Association of National Advertisers, 1975); and Russell D. Bowman, "MC's Second Annual Repo on Advertising and Promotion Expenditures," Marketing Communications, August, 1981.

A related stream of research uses sensitivity to promotions as a segmentation variable (Blattberg and Sen 1974, 1976; Blattberg, Peacock and Sen 1978). Blattberg and Sen (1976) segment on the basis of 1) brand loyalty (loyal or switcher), 2) type of brand preferred (national, private, both) and 3) price sensitivity (deal prone or not). Others have attempted to characterize the "deal-prone segment" with demographic and psychographic variables (Webster 1965; Massy, Frank and Lodahl 1968; Montgomery 1971; Blattberg, Buesing, Peacock and Sen 1978).

Another collection of papers considers the impact of promotional purchases on subsequent preferences among products. Rothschild and Gaidis (1981) draw on behavioral learning theory to suggest that well designed promotions can increase a brand's repurchase rate. Cotton and Babb (1978) found that post-promotion purchase rates for dairy products were generally higher than pre-promotion purchase rates. In contrast, Blattberg, Eppen and Lieberman (1981) rejected the premise that the function of promotions is to attract new customers. Shoemaker and Shoaf (1977) found that the probability of repurchasing a brand actually dropped if the previous purchase was on promotion. Scott (1976) and Dodson, Tybout and Sternthal (1978) propose self-perception theory as an explanation of this phenomenon. That theory suggests that the probability of repurchase is lowered because the individual is uncertain whether her selection of the promoted product is attributable to a liking for the purchased product or a desire to take advantage of the promotion. Because the effects noted in these studies are small and contradictory, we will not incorporate them in our model of market response.

Blattberg, Eppen and Lieberman (1981) directly address the motivation for promotions. Instead of assuming that promotions increase sales through brand switching, they propose that sales increase because consumers stockpile (buy

more units of the brand than usual). "[D]ealing occurs because retailers have higher inventory holding costs than some consumers. The retailer is motivated to take a reduction in sales revenue if the consumer is willing to carry some of the inventory. The consumer is willing to carry some inventory in return for a reduction in price." (Blattberg, Eppen and Lieberman 1981, p. 117)

In their theoretical model they assume that only one brand is available. Hence, brand switching is tautologically not an issue. They also suggest that the manufacturer's motive for offering the promotion to the retailer might also hinge on inventory holding costs. However, they do not explore the manufacturer's decision problem in any depth.

Following up on the stockpiling argument, Neslin, Quelch and Henderson (1983) examine the phenomenon of "accelerated purchasing". For the product class they investigate (bathroom tissue) they find that acceleration is predominantly exhibited in increased purchase quantity rather than shortened interpurchase times. Neslin and Shoemaker (1983) construct a model for evaluating the profitability of coupon promotions. Their model goes beyond the acceleration of purchases (the phenomenon of interest in Blattberg, Eppen and Lieberman, 1981) to include brand loyalty and repeat purchase effects (such as those suggested by Rothschild and Gaddis 1981 or Dodson, Tybout and Sternthal 1978). In an application of the model to a national brand of a low-cost personal care product, they found virtually no repeat purchase effect. They attributed the profitability of direct mail coupons relative to free standing inserts to the direct mailing's ability to attract more buyers not currently loyal to the brand. The purchase acceleration effect was negligible.

Consistent with the Neslin and Shoemaker (1983) findings and contrary to the Blattberg, Eppen and Lieberman (1981) arguments, Narasimhan (1982) proposes consumer switching as the motive for promotions. Based on a price

theoretic model he shows that more price elastic households are more intensive users of coupons. He constructs an optimization problem in which the manufacturer selects the product's price and the number and face value of coupons to be distributed in different media in order to maximize profits. While theoretically elegant, the formulation is, by the author's own admission, "a far cry from an operational model" (Narasimhan 1983, p. 28).

In this paper we develop an operational model to evaluate the profitability of consumer promotions. The model allows for deal-prone and non deal-prone consumers. Further, it allows the deal-prone consumers to respond to a promotion by stockpiling, switching or both. The model goes beyond Neslin and Shoemaker (1983) to evaluate the absolute profitability of a promotion rather than the relative profitability of several different promotional options.

We base our model on the segmentation scheme proposed by Blattberg and Sen (1976). We subdivide their "deal-prone" segments according to the consumers' response to promotions. We allow consumers three possible responses. They might stockpile their preferred brand when it is promoted. They might switch from their preferred brand to the promoted brand and buy a single unit. Or they might switch to the promoted brand and stockpile it. By modelling the profit implications of each of the possible consumer responses and then aggregating, we arrive at an expression for total market response to promotions.

We restrict ourselves to the consideration of the promotion of mature products. The purpose of promotions for new products is to stimulate trial and bring about awareness. The appropriate management of such a process is an important topic for research. However, we believe it to be a fundamentally different problem than the management of promotions for a mature product.

We will assume that manufacturers offer promotions directly to consumers. Insightful studies (e.g., Chevalier and Curhan 1975, 1976) have demonstrated that retailers do not pass along to consumers all promotions of which they avail themselves. We believe that the retailer's role in market response to promotions is an important one. We do not model that dimension in the interest of analytical tractability. However, we hold the incorporation of that phenomenon in the model to be an important direction for future research.

Finally, we abstract from the many possible promotional devices (e.g., coupons, cents-off labels, end-aisle displays, premiums) to a generic promotion. This generic promotion is characterized by a reduction of the manufacturer's profit per unit and a sudden addition to the brand's attractiveness. Deal prone consumers respond to the brand which has become suddenly more attractive by either stockpiling, switching or both.

The model is developed in detail in the next section.

MODEL

The model of consumer response to promotions will be presented in four phases. We begin by detailing the assumptions from which the model is built. The notation is then recapped. Next we develop analytical expressions for the expected gross profit to each segment. Finally, we form an appropriately weighted aggregate expression from the segment specific profit functions.

Assumptions

We begin (ASSUMPTION 1) by assuming that there is no effect of one purchase by a consumer on her subsequent product preferences. We consider the traditional time series effects, "learning"[5] (e.g., Keuhn, 1958) and "variety seeking" (e.g. McAlister and Pessemier, 1982), to be second order effects for this problem.

As mentioned earlier, we assume that manufacturers offer promotions directly to consumers (ASSUMPTION 2) and that a brand is considered to be "on promotion" if it has become "suddenly more attractive" (ASSUMPTION 3). This abstraction of promotions carries with it an assumption (ASSUMPTION 4) that promotions are of uniform intensity. A brand either is "suddenly more attractive" or it is not.

[TABLE 1 ABOUT HERE]

When a brand is not being promoted, the manufacturer receives a profit of M_1 per unit sold. When a brand is being promoted, the manufacturer receives a reduced profit M_2 ($M_2 < M_1$) per unit sold (ASSUMPTION 5).

The typical inter-purchase interval in this product class (i.e., the typical purchase cycle) is assumed known and constant (ASSUMPTION 6). Furthermore, it is assumed that each consumer averages one purchase from this product class during each purchase cycle (ASSUMPTION 7).

A critical assumption (ASSUMPTION 8) in this analysis is that those consumers who regularly purchase from this product class can be partitioned into seven segments based on their purchase behavior. In addition to these segments composed of regular consumers of the product class, we propose (ASSUMPTION 9) an eighth segment of consumers who would not make any purchase in the product class if there were no promotion.[6] Figure 2 illustrates this segmentation scheme. Assumptions 10-16 describe the assumed purchase behavior of consumers in the first seven segments.

[FIGURE 2 ABOUT HERE]

In the absence of promotions, loyal consumers always (ASSUMPTION 10) purchase their most preferred brand. Furthermore, we consider promotions within a single selling area so that the proportion of the population loyal to any particular brand is constant (ASSUMPTION 11). Given that there are z relevant competitive brands in this class, B_1, B_2,...,B_Z; we will let P_j represent the proportion of the population loyal to brand j. The vector $(P_1, P_2,...,P_Z)$, then, describes the share of loyal purchases going to each brand on those occasions when no brands are being promoted.

Those consumers that stockpile are assumed to purchase a fixed number of units, n, from a product class when they stockpile (ASSUMPTION 12). These consumers are assumed to deplete their current inventory from a product class before they make another purchase from that product class even if a promotion occurs(ASSUMPTION 13).

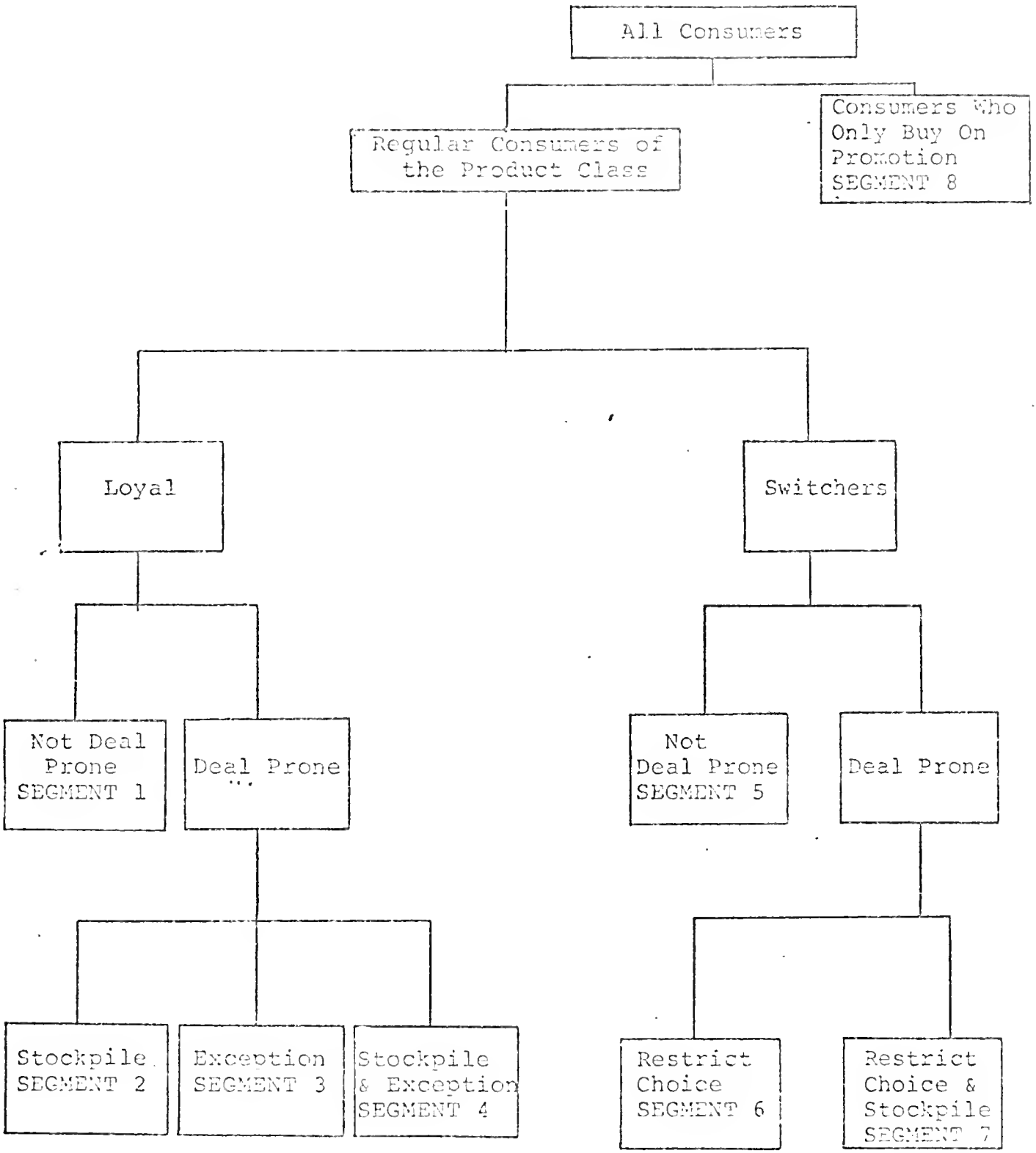
Deal Prone Loyal households might make an exception and purchase a less preferred, but promoted, brand when their most preferred brand is not being promoted. The share of such a consumer segment going to each promoted brand is proportional to that brand's share when no brand is promoted (ASSUMPTION 14). If B represents the subset of all brands on promotion at a particular point in time and if $B_i \epsilon B$, then within the Loyal--Deal Prone: Exception segment and the Loyal--Deal Prone: Stockpile and Exception segment the share of loyal purchases going to brand B should rise from P_i to $\frac{P_i}{\sum_{B_j \epsilon \hat{B}} P_j}$.

across all brands equals one, the sum of just those P_j's corresponding to brands being promoted ($B_j \epsilon \hat{B}$) must be less than one. Dividing P_i

Table 1: Assumptions of the Model

1. No effect of one purchase on preferences for subsequent purchases.
2. Manufacturer offers promotions directly to consumers.
3. "On Promotion" means "suddenly more attractive".
4. Promotions are of uniform intensity.
5. M_1 = regular profit. M_2 = promoted profit. $M_1 > M_2$.
6. Purchase cycle is known and constant.
7. Each household averages one purchase per purchase cycle.
8. The market can be partitioned into 8 segments.
9. There may be a segment of consumers who would not buy in the product class if there were no promotions.
10. Loyal, not deal prone households, always purchase their most preferred brand.
11. P_j's are constant across selling areas.
12. Stockpilers purchase n units when they stockpile.
13. Stockpilers deplete their inventory before making more purchases.
14. $$p = \frac{1}{\sum_{B_j \epsilon \hat{B}} P_j}$$
15. Switcher, not deal prone households, choose according to a multinomial process characterized by $(\pi_1, \pi_2, \ldots\ldots, \pi_z)$
16. Switchers are homogeneous.
17. $$k = \frac{1}{\sum_{B_j \epsilon \hat{B}} P_j}$$
18. α = P[B_1 is on promotion]
19. β = the largest value for α that doesn't degrade consumers' or retailer's perceptions of B_1's price.
20. β = P [Some brand other than B_1 is being promoted but B_1 is not]
21. k and p are known and constant.
22. $(S_1, S_2, \ldots, S_8)$ describes demand in segments 1 through 8.

FIGURE 2: SEGMENTATION SCHEME



by a number less than one results in a number greater than P_i.[7] We will denote this "coefficient of promotional leverage" among the Loyal segments as

$$p = \frac{1}{\sum_{B_j \epsilon \hat{B}} P_j} .$$

We assume that switchers are homogeneous (ASSUMPTION 15) and that they choose from the set of brands $(B_1, B_2, \ldots, B_Z)$ according to a vector of preference intensities for those brands, $(\pi_1, \pi_2, \ldots, \pi_Z)$ (ASSUMPTION 16). That is, π_1 of the time brand B_1 is chosen; π_2 of the time, B_2 is chosen; etc.[8]

Deal-prone switchers who restrict their choice to the set B of brands being promoted will increase their probability of choosing promoted brand $\hat{B}_i \epsilon B$ from π_i to (ASSUMPTION 17)

$$\frac{\pi_i}{\sum_{B_j \epsilon \hat{B}} \pi_j}$$

The quantity $\frac{1}{\sum_{B_j \epsilon \hat{B}} \pi_j}$ is greater than one (assuming that all brands are not being promoted). This "coefficient of promotional leverage" for Switchers,

$$k = \frac{1}{\sum_{B_j \epsilon \hat{B}} \pi_j} ,$$

is analogous to p, the "coefficient of promotional leverage" for Loyals.[9]

The analysis will consider the problem of promoting a single brand. Without loss of generality we can assume that the brand of interest is B_1. We take α as the probability that B_1 is on promotion when an

arbitrary consumer enters the market. α is taken as a constant across the planning period (ASSUMPTION 18). There exists some upper bound, $a \leq 1$, above which additions to the probability of encountering a promotion are not effective. This quantity is exogenously determined as a function of the consumers' and/or the retailers' perceptual processes. When α grows beyond a, consumers and/or retailers begin to consider the brand plus promotion as the regular product (e.g., Cracker Jacks and the prize, Duz detergent and a Cannon towel). We take this quantity, a, as a given (ASSUMPTION 19).

α, as just described, is the probability that the brand of interest, B_1, is being promoted. β will represent the probability that B_1 is not being promoted but that some brand (perhaps more than one) is being promoted. β is assumed (ASSUMPTION 20) to be constant across the planning period. We further assume that p and k, the coefficients of promotional leverage with Loyals and Switchers respectively, are known and constant across the planning period[10] (ASSUMPTION 21).

Finally, we assume that demand from each of the eight segments is known and constant across the planning period (ASSUMPTION 22). The vector $(S_1, S_2,\ldots,S_8)$ gives the number of consumers in each of the eight segments. Since (in ASSUMPTION 7) we assumed that each consumer buys an average of one unit per purchase cycle, knowing $(S_1, S_2,\ldots,S_8)$, is equivalent to knowing demand.

Notation

The notation to be used in developing the model includes:

M_1 = profit per unit when unpromoted

M_2 = profit per unit when promoted ($M_2 < M_1$)

n = the number of units of a promoted product purchased by members of those segments that stockpile (segments 2, 4, 7), $n \geq 1$

Z = the number of brands in the market

B = the set of all brands in the market
= $\{B_1, B_2, \ldots, B_Z\}$

B_1 = brand of interest in this analysis

$\hat{B}$ = set of brands being promoted when B_1 is promoted

$B_1 \varepsilon \hat{B} \subseteq B$

π_j = preference intensity for brand j, $\sum_{B_j \varepsilon B} \pi_j = 1$

P_j = proportion of "loyal" consumers that are loyal to brand B_j, $\sum_{B_j \varepsilon B} P_j = 1$

$p = \dfrac{1}{\sum_{B_j \varepsilon \hat{B}} P_j}$ = coefficient of promotional leverage with Loyals

$k = \dfrac{1}{\sum_{B_j \varepsilon \hat{B}} \pi_j}$ = coefficient of promotional leverage with Switchers

α = the probability that B_1 is being promoted

β = the probability that some brand is being promoted but B_1 is not being promoted

α = the largest probability of promoting B_1 that:

1) does not lower consumers' perception of B_1's regular price; and

2) stimulates a greater response from retailers than would be stimulated by a slightly smaller probability.

Manufacturer 1 = manufacturer of brand B_1

G_s = expected gross profit per household in segment s to manufacturer 1

Expected Gross Profit by Segment

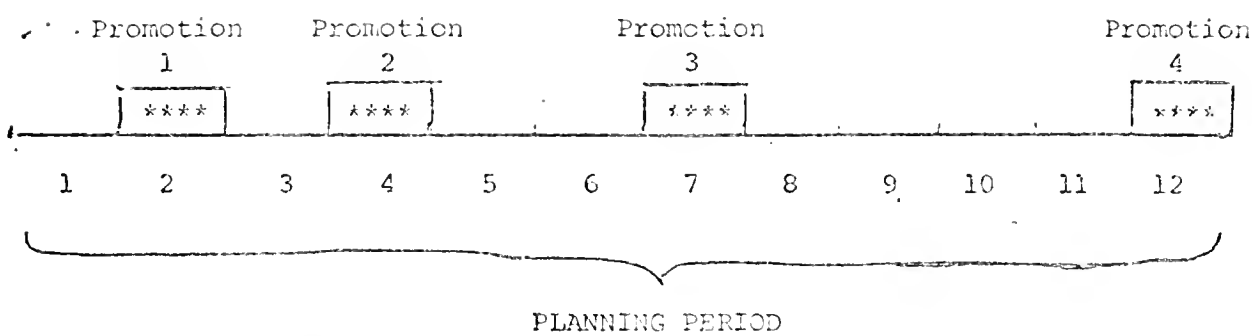
In order to calculate the expected gross profit from each segment, we choose to calculate the profit in an "expectd purchase cycle". To understand what we mean by that term, consider the promotion calendar depicted in Figure 3. That calendar displays manufacture 1's promotion plans for the 12 purchase cycles that make up a hypothetical planning period. Manufacturer 1 plans to promote B_1 in purchase cycles 2, 4, 7 and 12. This plan implies that B_1 will be on promotion 30% of the time.

Given that a manufacturer has no additional information (e.g., no knowledge of other manufacturers' promotion plans) when setting its promotion schedule, we have no reason to believe that the decision to promote in purchase cyles 2, 4, 7 and 12 is any better or worse than the decision to promote in cycles 1, 3, 6 and 11 or any other set of four purchase cycles. We can, therefore, characterize any promotion calendar by the percent of the planning period it allocates to promotional activity.

[FIGURE 3 ABOUT HERE]

In the modelling that follows, we assume that a consumer has come into the market at a random point in time. If the promotion calendar for B_1 dictates that it be on promotion α of the time, we assign probability α to the event that the consumer finds B_1 on promotion. We now consider the profit from a typical consumer in each of the segments during this "expected purchase cycle".

FIGURE 3: MANUFACTURER 1'S PROMOTION CALENDAR



SEGMENT 1: Loyal--Not Deal Prone

Consumers in this segment always buy their favorite brand. Their behavior is totally unaffected by promotions of their favorite brand or any other brand. P_1 of these consumers are loyal to brand B_1. With probability α they will find B_1 on promotion and their purchase will yield promoted profit M_2 to manufacturer 1 (the manufacturer of B_1). With probability $1-\alpha$ they will find that B_1 is not being promoted and their purchase will yield regular profit M_1 to manufacturer 1. Hence:

$$G_1 = P_1(\alpha M_2 + (1-\alpha)M_1) \qquad (1)$$

SEGMENT 2: Loyal--Deal Prone: Stockpile

Consumers in this segment only buy their favorite brand. If their favorite brand is being promoted, they will buy n units. Because these consumers stockpile, the purchase behavior in one period can affect purchase behavior in future periods. To calculate the expected gross profit for a typical period, one must consider purchase patterns for n periods and then take an average.

With probability α a consumer loyal to B_1 will discover that B_1 is being promoted. This consumer will buy n units of B_1 and then be out of the market for the next n-1 periods. Manufacturer 1 receives profit M_2 for each of the n units.

With probability $(1-\alpha)$ a consumer loyal to B_1 will discover that B_1 is not being promoted. This consumer will purchase one unit of B_1 for which manufacturer 1 will receive profit M_1. In the subsequent n-1 periods this consumer loyal to B_1 may or may not encounter B_1 on promotion. Our

best guess at the expected profit in those periods is G_2', the expected profit from a typical B_1 loyal consumer in this segment. We therefore arrive at the following recursive relationship for G_2':

$$G_2' = \alpha M_2 + (1-\alpha) \{ \frac{M_1}{n} + \frac{(n-1)}{n} G_2' \}$$

which implies that:

$$G_2' = \frac{1}{1 + (n-1)\alpha} (\alpha n M_2 + (1-\alpha)M_1).$$

Since P_1 of this segment are loyal to B_1, we would expect gross profit from the segment as a whole to be given by $G_2 = P_1 G_2'$, or

$$G_2 = \frac{P_1}{1 + (n-1)\alpha} (\alpha n M_2 + (1-\alpha)M_1) \qquad (2)$$

To interpret equation (2), recall that those consumers with existing inventories are assumed to be invulnerable to promotions. At any given point in time only $\frac{1}{1 + (n-1)\alpha}$ of the consumers loyal to B_1 are expected to be vulnerable.[11] Hence, the proportion of Segment 2 vulnerable to a promotion of B_1 is less than P_1, and is, in fact, equal to $\frac{P_1}{1+(n-1)\alpha}$. The profit that can be expected from each vulnerable consumer in Segment 2 is $\alpha n M_2$ (profit if a promotion is encountered, nM_2, times the probability a promotion is encountered, α) plus $(1-\alpha)M_1$ (profit if no promotion is encountered, M_1, times the probability that no promotion will be encountered.)

SEGMENT 3: Loyal--Deal Prone: Exception

Consumers in this segment buy their favorite brand unless that favored brand is not on promotion and some competitive brand is being promoted. In this case these consumers make an exception and purchase one of the brands being promoted.

If more than one brand is being promoted, those consumers loyal to unpromoted brands will have to choose from the set of promoted brands. Assuming that the proportion of the consumers loyal to a particular brand is indicative of its relative attractiveness, the total number of consumers that will choose B_i is

$$P[\text{choose } B_i \in \hat{B} \mid \hat{B} \text{ promoted}] = \frac{P_i}{\sum_{B_j \in \hat{B}} P_j} = P_i p \quad \text{where } p = \frac{1}{\sum_{B_j \in \hat{B}} P_j} .$$

With probability α, these consumers will discover B_1 on promotion and pP_1 of them will buy B_1 providing manufacturer 1 with M_2 per unit. With probability β these consumers will discover some brands other than B_1 on promotion. They will purchase one of the promoted brands yielding 0 to manufacturer 1.

With probability $1-\alpha-\beta$ these consumers will find no brands on promotion. The P_1 of these consumers loyal to B_1 will purchase B_1 yielding profit M_1 per unit to manufacturer 1.

$$G_3 = P_1(\alpha p M_2 + (1-\alpha-\beta)M_1) \tag{3}$$

SEGMENT 4: Loyal--Deal Prone: Stockpile and Exception

Consumers in this segment buy their favorite brand when no brands are being promoted. If their favorite brand is on promotion, they buy n units of

that brand. If their favorite brand is not being promoted and some competitive brands are being promoted these consumers make an exception and purchase n units of one of the brands being promoted.

Since stockpiling occurs, we must again consider purchase behavior over n periods. With probability α these consumers will find B_1 on promotion. pP_1 of these consumers will buy n units each yielding profit M_2 per unit to manufacturer 1. These pP_1 consumers will be out of the market for the next n periods.

With probability β these consumers find some brand other than B_1 on promotion and B_1 not on promotion. In this case the promoted brands will be stockpiled and manufacturer 1 will receive 0 for n periods.

With probability $1-\alpha-\beta$ these consumers will find no brands on promotion. The P_1 of these consumers loyal to B_1 will buy 1 unit each of B_1 at profit M_1. In the subsequent n-1 periods these consumers may or may not find B_1 or some other brand on promotion. Manufacturer 1 would expect to receive margin G_4 from these consumers in each of those n-1 periods. Recursively, then:

$$G_4 = \alpha\, pP_1M_2 + (1-\alpha-\beta)\left(P_1M_1\left(\frac{1}{n}\right) + \left(\frac{n-1}{n}\right)G_4\right)$$

or

$$G_4 = \frac{P_1}{1+(n-1)(\alpha+\beta)}\left(n\alpha pM_2 + (1-\alpha-\beta)M_1\right) \qquad (4)$$

As with equation (2), we can interpret equation (4) as saying that $\frac{1}{1+(n-1)(\alpha+\beta)}$ of the members of segment 4 are expected to be in the market at any given time. With probability α, pP_1 of those vulnerable consumers will buy n units of B_1 on promotion. With probability $1-\alpha-\beta$, P_1 of those vulnerable consumers will buy one unit of B_1 at the regular price.

SEGMENT 5: Switcher--Not Deal Prone

Consumers in this segment choose from the brands available according to a multinomial process characterized by the vector $(\pi_1, \pi_2, \ldots, \pi_Z)$. π_1 represents the probability of selecting brand B_1; π_2, the probability of selecting B_2, etc. Their behavior is totally unaffected by promotions. On any given choice occasion, we would expect π_1 of these consumers to select B_1. With probability α, B_1 will be promoted yielding profit M_2 to manufacturer 1. With probability $1-\alpha$, B_1 will not be promoted yielding profit M_1 to manufacturer 1. Expected profit per consumer in this segment is, then:

$$G_5 = \pi_1(\alpha M_2 + (1-\alpha)M_1) \qquad (5)$$

SEGMENT 6: Switcher--Deal Prone: Restrict Choice

Consumers in this segment choose from the brands available according to a multinomial process characterized by the vector $(\pi_1, \pi_2, \ldots, \pi_Z)$ so long as no brand is being promoted. When a subset $\hat{B}$ of B is being promoted, these consumers purchase one unit and restrict their choice to the set $\hat{B}$. The probability of choosing brand B_1 in $\hat{B}$ is

$$P[\text{choose } B_1 \epsilon \hat{B} \mid \hat{B} \text{ promoted}] = \frac{\pi_1}{\sum_{B_j \epsilon \hat{B}} \pi_j} = k\pi_1$$

With probability α, these consumers will find B_1 on promotion and $k\pi_1$ of them will buy B_1, providing manufacturer 1 with M_2 per unit. With probability β these consumers will find some brands other than B_1 on promotion. These consumers will purchase one of the promoted brands yielding

0 to manufacturer 1. With probability $1-\alpha-\beta$ these consumers will find no brands on promotion. π_1 of the consumers will purchase B_1 yielding profit M_1 per unit to manufacturer 1.

$$G_6 = \pi_1(\alpha\, k\, M_2 + (1-\alpha-\beta)M_1) \tag{6}$$

SEGMENT 7: Switcher---Deal Prone: Restrict Choice and Stockpile

Consumers in this segment choose from the brands available according to a multinomial process characterized by the vector $(\pi_1, \pi_2, \ldots, \pi_Z)$ so long as no brand is being promoted. When a subset $\hat{B}$ of B is being promoted, these consumers purchase n units and restrict their choice to the set $\hat{B}$.

Since stockpiling occurs, we must again consider purchase behavior over n periods. With probability α these consumers will find B_1 on promotion. $k\pi_1$ of these consumers will buy n units each yielding profit M_2 per unit to manufacturer 1. These $k\pi_1$ consumers will be out of the market for the next n-1 periods.

With probability β these consumers find some brands other than B_1 on promotion. In this case the promoted brand will be stockpiled and manufacturer 1 will receive 0 for n periods.

With probability $1-\alpha-\beta$ these consumers will find no brands on promotion. π_1 of them will buy one unit each of B_1 at profit M_1 per unit. In each of the subsequent n-1 periods manufacturer 1 expects margin G_7 from these consumers.

and
$$G_7 = \alpha k \pi_1 M_2 + (1-\alpha-\beta)\left(\pi_1 M_1 \left(\frac{1}{n}\right) + \left(\frac{n-1}{n}\right) G_7\right)$$

$$G_7 = \frac{\pi_1}{1+(n-1)(\alpha+\beta)} \left(n\, \alpha\, k\, M_2 + (1-\alpha-\beta)M_1\right) \tag{7}$$

Equation (7) can be interpreted as saying that $\frac{1}{1+(n-1)(\alpha+\beta)}$ of the consumers in segment 7 are expected to be vulnerable to a promotion at any given point in time. With probability α, $k\pi_1$ of the vulnerable consumers will buy n units of B_1 on promotion. With probability $1-\alpha-\beta$, π_1 of the vulnerable consumers will buy one unit of B_1 off promotion.

Segment 8: Consumers Who Buy Only on Promotion

Consumers in this segment would never buy in the product class if there were no promotions. Therefore no sales are ever made to these consumers at margin M_1. When these consumers observe a promotion, they choose from the promoted products according to a set of preference intensities like those described for deal prone switchers. The expected profit from the non-stockpiling consumers in this segment is:

$$G_{8a} = \pi_1 \alpha k M_2 \tag{8a}$$

For those consumers who do stockpile, the expected profit

$$G_{8b} = \frac{\pi_1}{1+(n-1)(\alpha+\beta)} \quad (\alpha k n M_2) \tag{8b}$$

Generic Gross Profit Function

Examination of equations (1)-(8) shows a relationship among the segment profit functions. A more general statement, of which equations (1)-(8) are special cases, is given in the generic profit function, G_s:

$$G_s = \frac{\pi_s}{1+(n_s-1)(\alpha+\beta_s)} \quad (\alpha k_s n_s M_2 + (1-\alpha-\beta_s)M_{1s}) \tag{9}$$

where

Segment Number (s)	Segment Description	Values in Segments for $s\pi$	$s\beta$	n_s	k_s	M_{1s}
1	Loyal--Not Deal Prone	P_1	0	1	1	M_1
2	Loyal--Deal Prone: Stockpile	P_1	0	n	1	M_1
3	Loyal--Deal Prone: Exception	P_1	β	1	p	M_1
4	Loyal--Deal Prone: Stockpile & Exception	P_1	β	n	p	M_1
5	Switcher--Not Deal Prone	π_1	0	1	1	M_1
6	Switcher--Deal Prone: Restrict Choice	π_1	β	1	k	M_1
7	Switcher--Deal Prone: Restrict & Stockpile	π_1	β	n	k	M_1
8a	Buy Only on Promotion--No Stockpiling	π_1	0	1	k	0
8b	Buy Only on Promotion and Stockpile	π_1	0	n	k	0

Total Profit

The generic profit function just developed describes the expected gross profit per person in each of the segments. The number of consumers in each of the segments is given by the vector $(S_1, S_2, \ldots, S_8)$. Therefore, total expected gross profit, G_T, is given by:

$$G_T = \sum_{s=1}^{8} S_s G_s = \sum_{s=1}^{8} \frac{S_s \pi_s}{1 + (n_s - 1)(\alpha + \beta_s)} (\alpha k_s n_s M_2 + (1-\alpha-\beta_s)M_{1s}) \qquad (10)$$

Before considering the question of whether the manufacturer should promote or not, we consider a generalization of the model just developed.

Consumers With Inventory Respond to Promotions

Recall that ASSUMPTION 13 held that consumers deplete their current inventory of a product before they repurchase from that class. It might happen that consumers do not repurchase at the regular price so long as they hold inventory. They might, however, be lured back into the market with another promotion. On such a purchase occasion, the consumer would buy just enough of the product to refill her stockpile. That is, if the consumer stockpiles n units when her inventory is completely empty, she will stockpile $n-n_o$ units when she encounters a promotion while possessing an inventory of n_o units.

Figures 4A and 4B display the implications of these two policies for a consumer loyal to B_1 who can accommodate at most 3 units of inventory (n = 3). For this example, we assume that manufacturer 1 uses the promotion calendar depicted in Figure 3 (i.e., B_1 is on promotion in purchase cycles 2, 3, 7 and 12). Under either policy, the consumer purchases one unit of B_1 in the first purchase cycle and consumes that unit during the cycle. Either policy would also have the consumer stock up to 3 units of inventory when she encounters the promotion in purchase cycle 2. This inventory would be consumed through cycles 2 and 3 since no further promotins are encountered.

[FIGURE 4 ABOUT HERE]

During purchase cycle 4, the two policies diverge. Under ASSUMPTION 13, the consumer ignores the promotion and continues to deplete her inventory (See Figure 4A). Under the relaxation of ASSUMPTION 13, the consumer takes

advantage of the promotion in purchase cycle 4 to replenish her inventory. The difference between the two policies boils down to the fact that, under ASSUMPTION 13, consumers ignore some promotions (those that occur when the consumer has inventoried products). Under the relaxation, consumers avail themselves of all promotions.

Consumer Inventorying Modelled as a Renewal Process

To understand the implications of consumers responding to all promotions, consider the renewal process (Cinlar, 1975) that begins when a consumer makes her first promoted purchase and ends when she has completely exhausted her inventory and made a single, non-promoted purchase.

To model this behavior, we assume that each manufacturer's promotions occur according to a Poisson process. (That is, the number of promotions between time t and time t + s depends only on s. Further, the number of promotions between time t and time t + s is independent of the number of promotions between time 0 and time t. The times between a manufacturer's promotions can, therefore, be represented by independent and identically distributed exponential random variables). With α = the average number of promotions offered by the manufacturer per purchase cycle, the cumulative distribution fuction for t, the time between two promotions is

$$1 - e^{-\alpha t}, \quad t \geq 0$$

To be consistent with this formulation, we must reconceptualize the promotion calendar. In Figure 3, we depicted promotions as events which extended for an entire purchase cycle. Further, consumers were implicitly assumed to make their purchase decisions at the beginning of each purchase

FIGURE 4: CONSUMER'S INVENTORY PATTERNS UNDER ASSUMPTION 13

AND UNDER THE RELAXATION OF ASSUMPTION 13

a = Percent of time on promotion = 30%

n = 3

Figure 4A:
CONSUMER'S INVENTORY
(Ignoring promotions when she has inventory)

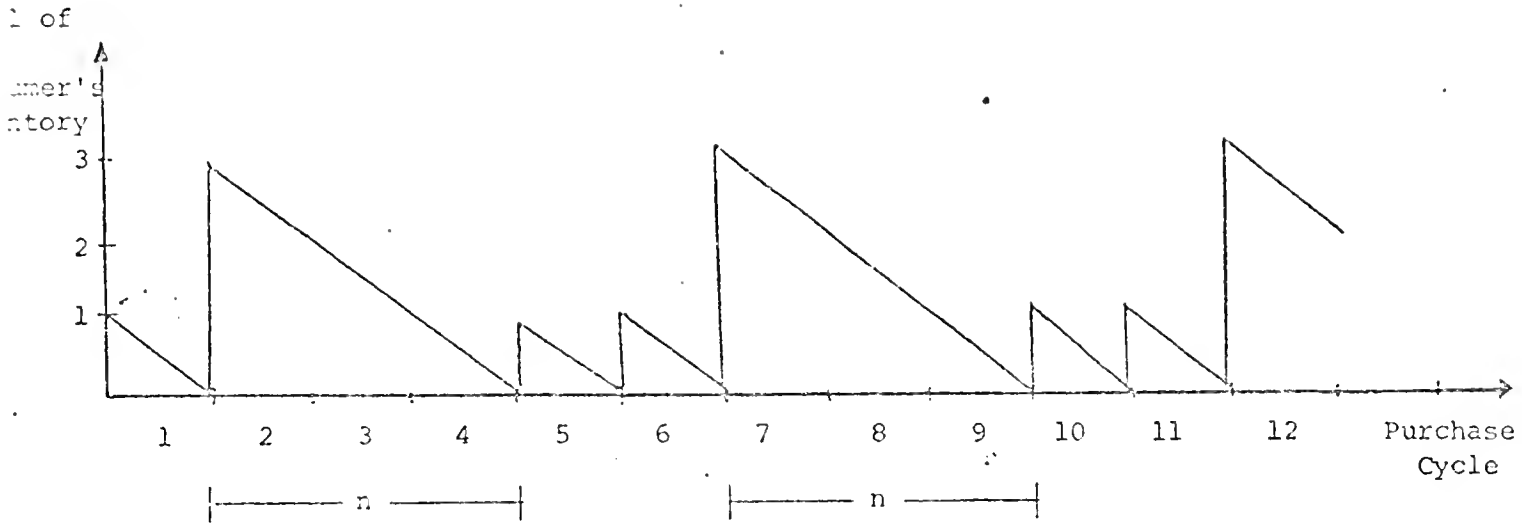
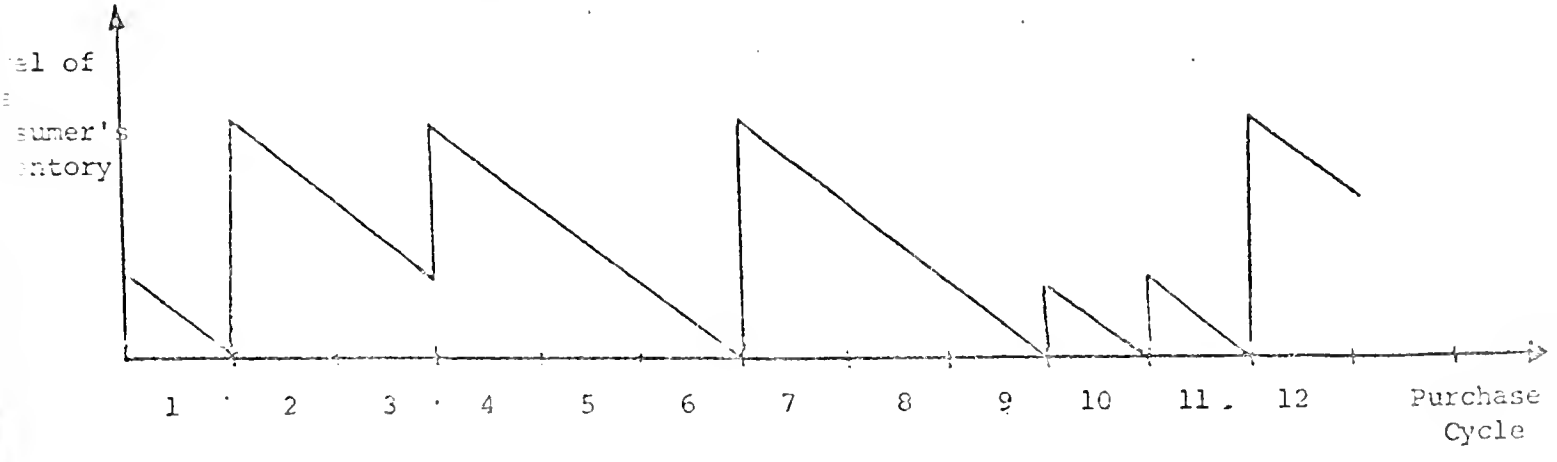


Figure 4B:
CONSUMER'S INVENTORY
(Restock inventory when a promotion occurs)



cycle. With the Poisson process framework, promotions are reduced to single points in time and consumers are assumed to be continuously shopping. Figure 5 translates the promotion calendar from Figure 2. Figure 6 shows the inventory pattern that would result from those promotions for a consumer loyal to B_1 who took advantage of all promotions to refill her stockpile.

[FIGURE 5 AND 6 ABOUT HERE]

If we assume that each manufacturer's pattern of promotion is independent of all other manufacturer's promotions, the pattern of promotions in the product class as a whole is also a Poisson process. The time between any two promotions in the product class is exponentially distributed with cumulative distribution function $1 - e^{z\alpha t}$ $t, \geq 0$, where z is the total number of brands in the product class.

Figure 7 represents hypothetical promotional calendars for brands B_2, B_3, and B_4 similar to that for brand B_1 in Figure 5. If these brands composed an entire product class, the calendar of promotional events for that class would be given by the bottom graph in Figure 7. Figure 8 illustrates the inventory pattern that a switcher who took advantage of all promotions to refill her stockpile would experience.

[FIGURES 7 AND 8 ABOUT HERE]

Notice that, in this case, <u>every</u> purchase made by this consumer was on promotion. The "lifetime" of the renewal process that begins when a promoted purchase is made must be nearly infinite here. This despite the fact that no brand is promoted more than 30% of the time. Furthermore, in this case it happens that no promotions at all are offered in purchase cycles 5 and 11.

Percent of Purchases Made on Promotin

We now consider the percent of purchases made by deal prone switchers on promotion and the sensitivity of that percentage to several parameters.

[FIGURE 9 ABOUT HERE]

From Figure 9c, we see that virtually all purchases are made on promotion when 8 brands promote. This holds even if no stockpiling occurs ($n = 1$) and even if brands only promote infrequently ($\alpha = 20\%$).

When only 4 brands promote (Figure 9b), promoted purchases grow beyond 50% of total purchases for small promotion frequencies ($\alpha = 20\%$) even without stockpiling ($n = 1$). When stockpiling does occur ($n = 2$ or 4), the percent of purchases on promotion approaches 100.

For the case in which only 1 brand promotes (the first brand in a category to promote or a brand promoting to loyal consumers), the percent of purchases on promotion is much lower. When there is no stockpiling ($n = 1$), only 50% of the purchases are on promotion even when promotions occur 70% of the time. This happens because of the assumption that the times between promotions are distributed exponentially. It could easily happen that several promotions occur in rapid succession followed by a long period with no promotions. The consumer's inventory would hardly drop at all during the frequent promotions. Hence, she would buy very few units (perhaps only partial units) on promotion during that time. During the stretch of time that no promotions occur, the consumer would buy at the non-promoted price.

[TABLE 2 ABOUT HERE]

FIGURE 6: B_1-LOYAL CONSUMER'S INVENTORY WHEN B_1'S PROMOTIONS OCCUR ACCORDING TO A POISSON PROCESS AND CONSUMERS TAKE ADVANTAGE OF ALL PROMOTIONS TO RESTOCK

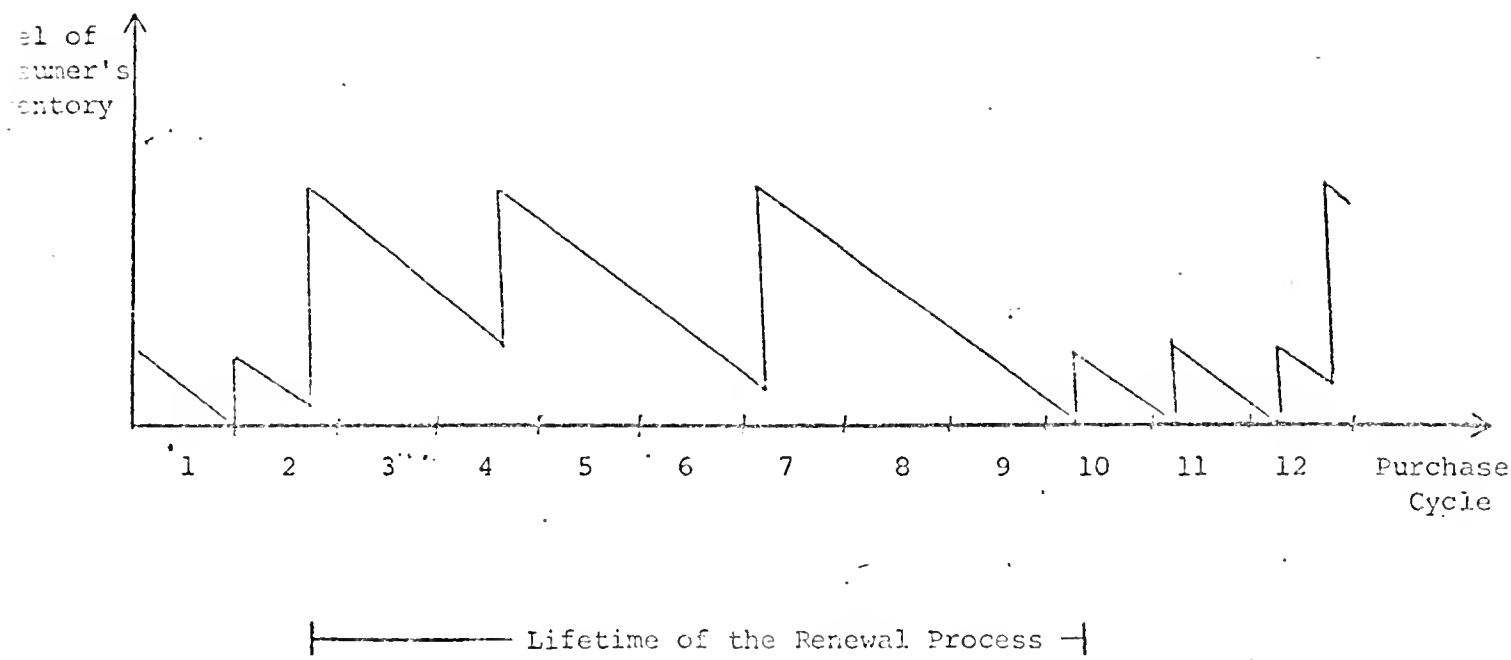


FIGURE 5: PROMOTION CALDENDAR WHEN PROMOTIONS ARE REPRESENTED AS SINGLE POINTS IN TIME

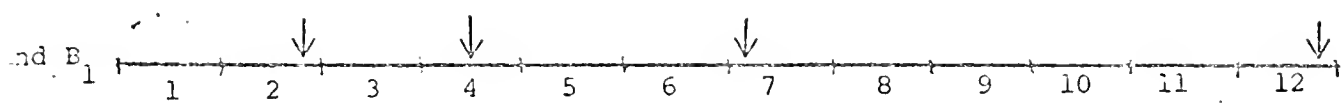


FIGURE 7: PROMOTION CALENDARS FOR MANUFACTURERS 2, 3 AND 4 AND FOR THE PRODUCT CLASS AS A WHOLE

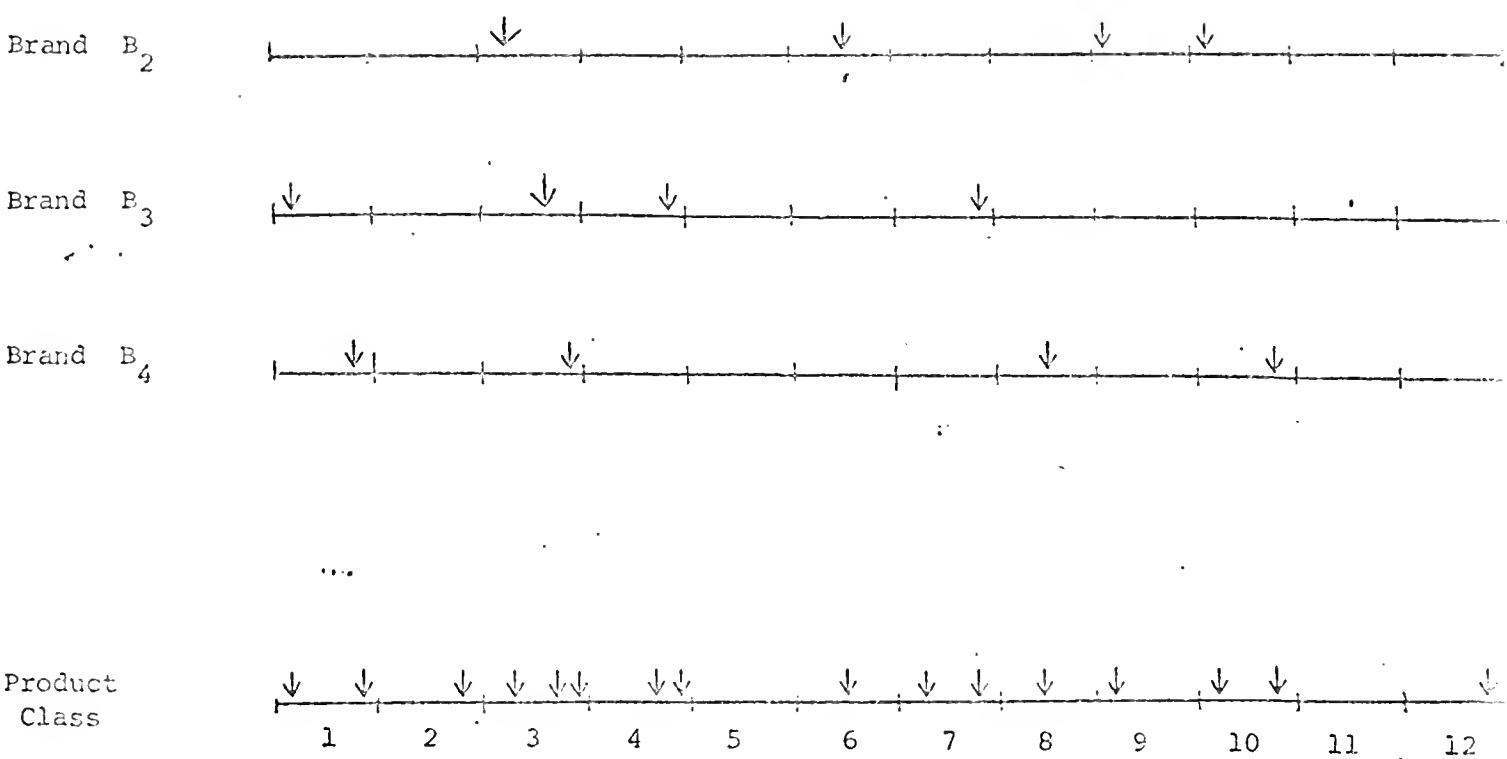


FIGURE 8: SWITCHER'S INVENTORY WHEN PRODUCT CLASS PROMOTIONS OCCUR ACCORDING TO A POISSON PROCESS AND CONSUMERS TAKE ADVANTAGE OF ALL PROMOTIONS TO RESTOCK

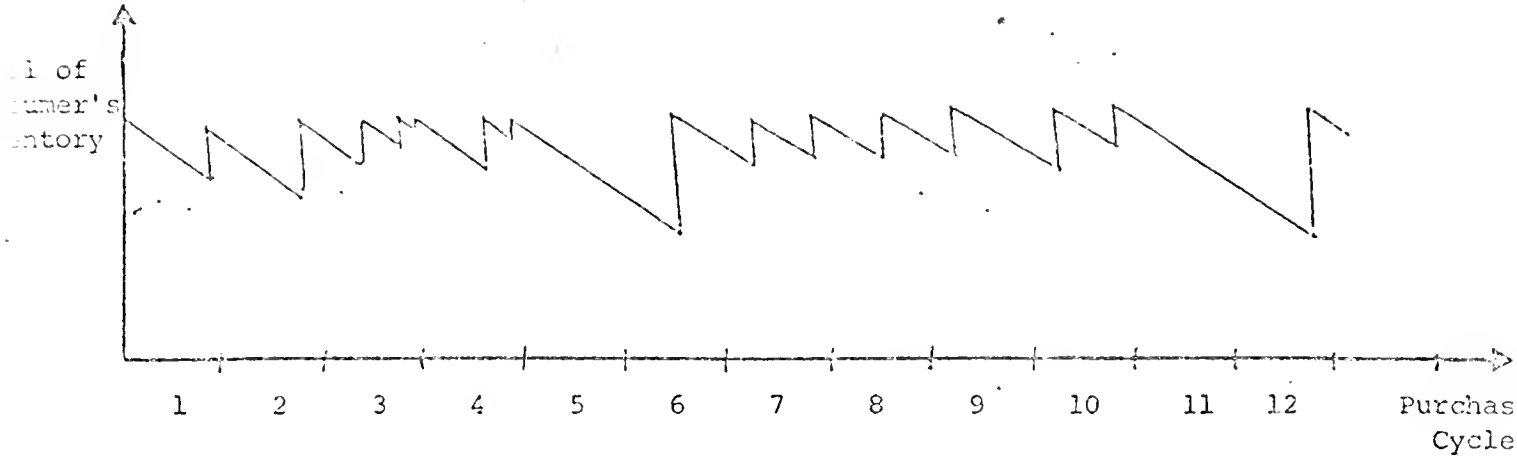


Figure 9A: 1 BRAND PROMOTES

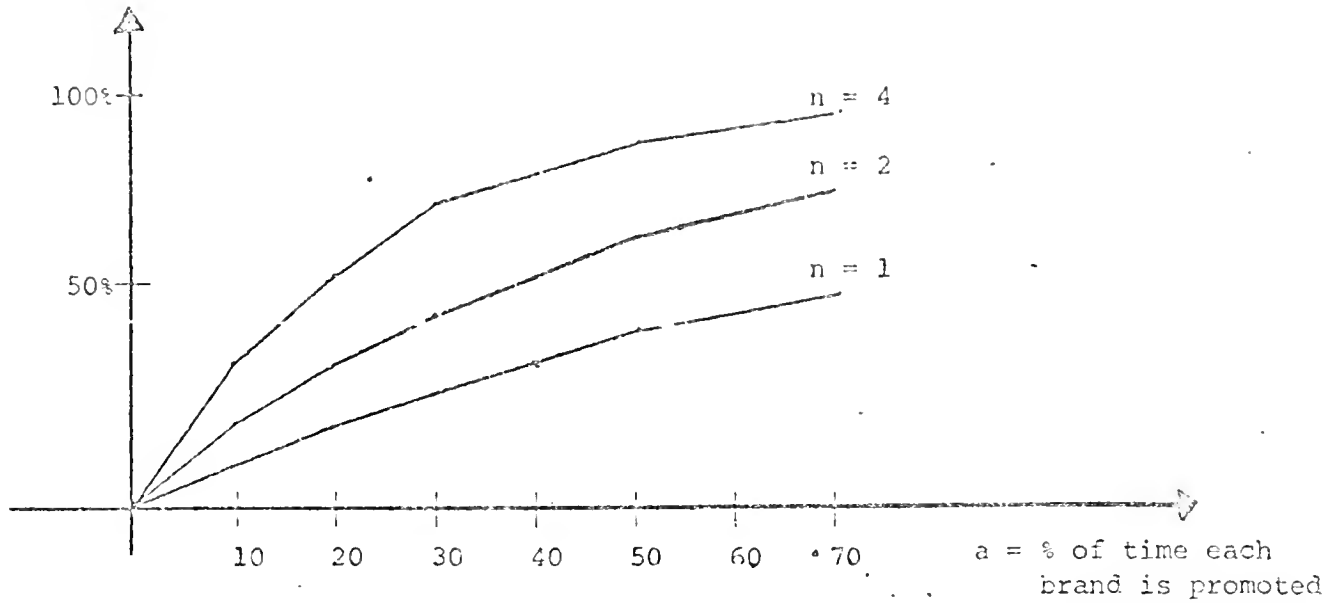


Figure 9B: 4 BRANDS PROMOTE

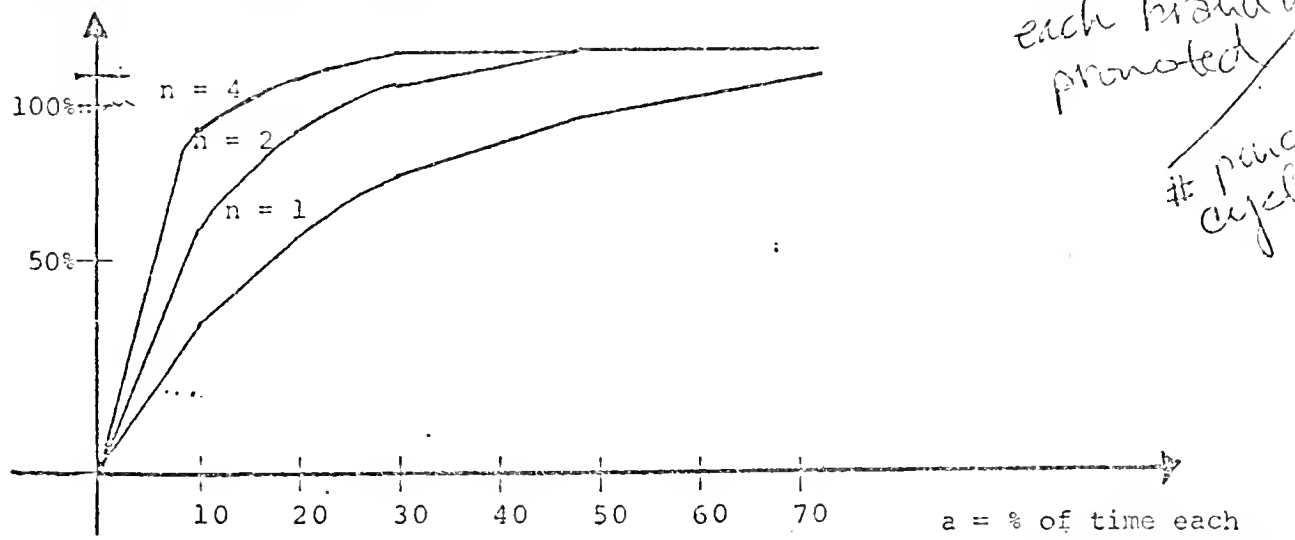


Figure 9C: 8 BRANDS PROMOTE

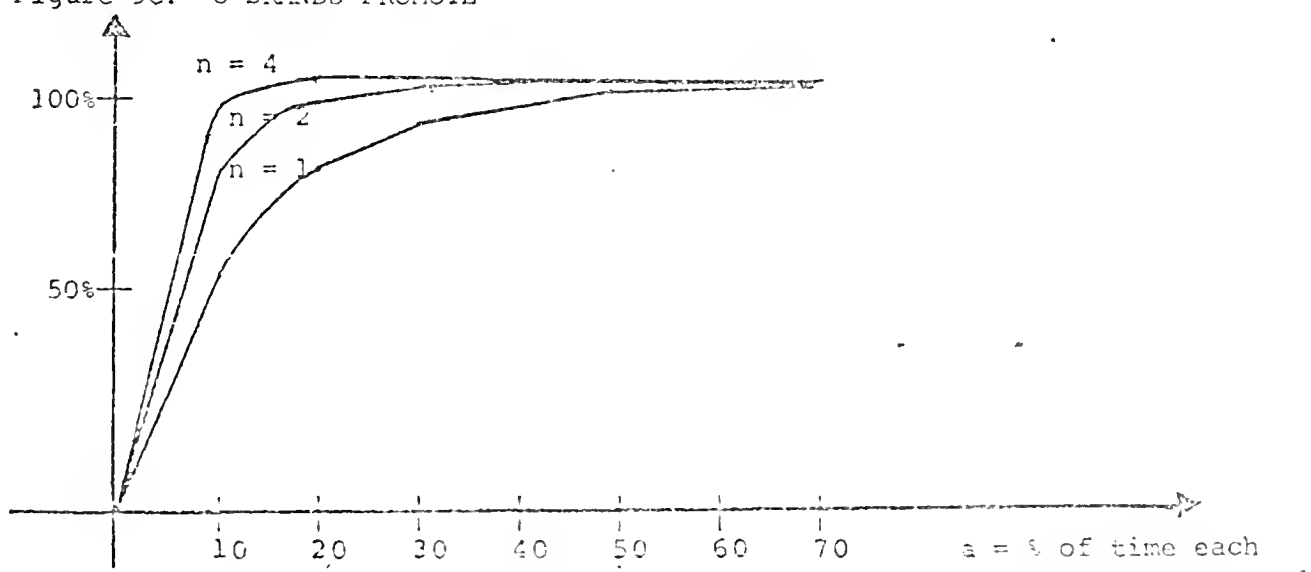


Table 2: Percent of Purchases on Promotion as a Function of z, n, and a

(1) Number of Competitors =z	(2) Number of Units Stock-piled = n	(3) % of Time a Brand is on Promotion =a	(4) Lifetime of the Renewal Process =L	(5) Expected # of Purchase Cycles till next Renewal =1/(a)	(6) Proportion of Purchases on Promotion = $\frac{L}{L+1/(a)}$
1	1	.1	1.00	10.00	.09
1	1	.2	1.10	5.00	.18
1	1	.3	1.17	3.33	.26
1	1	.5	1.38	2.00	.41
1	1	.7	1.44	1.43	.50
1	2	.1	2.20	10.00	.18
1	2	.2	2.45	5.00	.33
1	2	.3	2.73	3.33	.45
1	2	.5	3.44	2.00	.63
1	2	.7	4.36	1.43	.75
1	4	.1	4.90	10.00	.33
1	4	.2	6.15	5.00	.55
1	4	.3	7.73	3.33	.70
1	4	.5	12.76	2.00	.86
1	4	.7	22.08	1.43	.94
4	1	.1	1.23	2.50	.33
4	1	.2	1.54	1.25	.55
4	1	.3	1.93	.83	.70
4	1	.5	3.19	.50	.86
4	1	.7	5.56	.36	.94
4	2	.1	3.05	2.50	.55
4	2	.2	4.94	1.25	.80
4	2	.3	9.32	.83	.92
4	2	.5	26.80	.50	.98
4	2	.7	96.99	.36	.99+

TABLE 3: CONDITIONS FOR PROFITABILITY BY SEGMENT

Segment Number	Segment Description	Condition for Profitability	
1	Loyal Not Deal Prone	$M_2 - M_1 > 0$	(Never True)
2	Loyal Deal Prone: Stockpiler	$M_2 - M_1 > 0$	(Never True)
3	Loyal Deal Prone: Exception	$pM_2 - M_1 > 0$	
4	Loyal Deal Prone: Stockpiler and Exception	$p(1 + \beta(n-1)M_2 - M_1 > 0$	
5	Switcher Not Deal Prone	$M_2 - M_1 > 0$	(Never True)
6	Switcher Deal Prone: Restrict Choice	$kM_2 - M_1 > 0$	
7	Switcher Deal Prone: Restrict Choice and Stockpile	$k(1 + \beta(n-1)) M_2 - M_1 > 0$	
...			
8_a	Buy Only on Promotion: No Stockpiling	$kM_2 > 0$	(Always True if $M_2 > 0$)
8_b	Buy Only on Promotion: Stockpile	$kM_2 > 0$	(Always True if $M_2 > 0$)

Table 2, Cont'd.

(1) Number of Competitors =z	(2) Number of Units Stock-piled = n	(3) % of Time a Brand is on Promotion =a	(4) Lifetime of the Renewal Process =L	(5) Expected # of Purchase Cycles till next Renewal =1/(a)	(6) Proportion of Purchases on Promotion = $\frac{L}{L+1/(a)}$
4	4	.1	9.88	2.50	.80
4	4	.2	29.41	1.25	.96
4	4	.3	100.02	.83	.99
4	4	.5	1,489.98	.50	.99+
4	4	.7	26,326.96	.36	.99+
8	1	.1	1.53	1.25	.55
8	1	.2	2.49	.63	.80
8	1	.3	4.21	.42	.91
8	1	.5	13.40	.25	.98
8	1	.7	48.50	.18	.99+
8	2	.1	4.94	1.25	.80
8	2	.2	14.82	.63	.96
8	2	.3	50.61	.42	.99
8	2	.5	744.99	.25	.99+
8	2	.7	13,163.48	.18	.99+
8	4	.1	29.41	1.25	.96
8	4	.2	378.54	.63	.99+
8	4	.3	6,201.21	.42	.99+
8	4	.5	2,221,527.60	.25	.99+
8	4	.7	9.626×10^8	.18	.99+

Table 2 contains the detailed information used to produce Figure 9. First, following Cinlar (1975, p. 292), we calculate the "lifetime" of the renewal process, L, as

$$L = \frac{1}{Z\alpha} \left(e^{Z\alpha n} - 1\right).$$

This expression tells us how long we expect a consumer to make consecutive purchases on promotion. (Since the consumer is assumed to use one unit per purchase cycle, L is also the expected number of consecutive purchases on promotion). Column 4 reports the value of L for different numbers of competitors (Z in colume 1), different numbers of units stockpiled (n is column 2), and different percents of time each brand is promoted (α in column 3). Notice that L increases with each of the three parameters (z, n and α) and becomes very large when any pair of parameters becomes large.

Column 5 reports the expected number of purchase cycles that will elapse between the end of one renewal (when inventory is exhausted and a non-promoted purchase is made) and the beginning of the next renewal (when the next promotion is encountered). Because of the "memoryless" property of exponential inter-promotion times (Cinlar, 1975), the expected time until the next promotion is always $1/(z\alpha)$.

Column 6 reports the proportion of purchases made on promotion. This is calculated by dividing the expected number of purchases made on promotion (L) by the sum of expected promoted purchases (L) and unpromoted purchases ($1/(z\alpha)$).

Updating the Expected Profit Function

Just as we had to consider a consumer's purchase behavior for n periods when stockpiling was allowed in the original model, we must now consider purchase behavior for L periods. When a consumer makes a purchase on promotion, we expect that the consumer will not make a non-promoted purchase for the next L - 1 purchase cycles. If the first promoted purchase is of B_1 and n units are stockpiled, then B_1 will get n purchases at profit M_2 per unit. The remaining L - n purchases might go to any of the brands that offer promotions. We expect $\alpha/(\alpha + \beta)$ of those purchases to go to B_1. (B_1 is on promotion α of the time. Other brands are on promotion β of the time.) Keeping this in mind, we suggest that:

$$G_s = \alpha \left\{ \left(\frac{n_s}{L_s}\right) M_2 + \left(\frac{L_s - n_s}{L_s}\right) \left(\frac{a}{a + \beta_s}\right) M_2 \right\} \pi_s k_s$$

$$+ \beta_s \left\{ \left(\frac{n_s}{L_s}\right) 0 + \left(\frac{L_s - n_s}{L_s}\right) \left(\frac{\alpha}{\alpha + \beta_s}\right) M_2 \right\} \pi_s k_s$$

$$+ (1-\alpha-\beta_s) \left\{ \left(\frac{1}{L_s}\right) M_{1s} + \left(\frac{L_s - 1}{L_s}\right) G_s \right\}$$

or

$$G_s = \frac{\pi_s}{1 + (L_s - 1)(\alpha + \beta_s)} \left\{ L_s k_s \alpha M_2 + (1 - \alpha - \beta_s) M_{1s} \right\}$$

where

$$L_s = \frac{1}{Z\alpha} \left(e^{Z\alpha n_s} - 1\right).$$

In this section we have relaxed the assumption that consumers ignore promotions so long as they have inventoried product. We modelled the occurance of promotions as a Poisson process and allowed consumers to respond to all promotions by refilling their stockpiles. The number of consecutive promoted purchases was shown to be the "lifetime" of the related renewal process. We further showed that, in many cases, this paradigm suggests that virtually all purchases will be made on promotion. Finally, we showed that one need only substitute L_s for n_s in the generic profit function to estimate expected profits.

Since both consumer stockpiling policies (ASSUMPTION 13: ignore promotions when there exists a stockpile and RELAXATION OF ASSUMPTION 13: take advantage of all promotions) probably occur, we could subdivide all segments that stockpile. This would bring the total number of segment specific models up to eleven. We decided, for expositional simplicity, to stay with the eight previously defined segments and the n_s notation. When actually applying this model, however, consumer inventorying policies should be examined and all relevent behaviors modelled.

In the next section, we consider the question of whether or not a manufacturer should offer promotions. We base our arguments on the model developed in equation 10. The extension of these arguments to include the stockpiling behavior described in this section is straightforward. We forego that presentation in the interest of expositional brevity.

IS PROMOTION PROFITABLE?

We begin by considering the profitability of promoting to each of the segments in isolation. These conditions are then aggregated to consider the profitability of promoting to the market as a whole.

To establish the attractiveness of promoting to any of the segments, we take the derivative of the generic segment profit function with respect to α. If that derivative is positive then the gross profit to manufacturer 1 increases as the probability of promoting B_1 increases. More promotion is better. If that derivative is negative then the gross profit to manufacturer 1 decreases as the probability of promoting B_1 increases. Less promotion is better. The generic derivative is:

$$\frac{\partial G_s}{\partial \alpha} = \frac{n_s \pi_s}{[1 + (n_s - 1)(a + \beta_s)]^2} \{k_s[1 + \beta_s(n_s - 1)]M_2 - M_{1s}\} \qquad (11)$$

Since $\frac{n_s \pi_s}{[1 + (n_s - 1)(a + \beta_s)]^2}$ is always positive, the sign of the derivative is determined by the sign of $k_s[1+\beta_s(n_s-1)]M_2-M_{1s}$. If this quantity is positive for a segment then the derivative is positive for that segment and it is profitable to promote to it.

The precise condition which must hold for each segment is a function of k_s, β_s and n_s. These conditions are reported in Table 3.

[TABLE 3 ABOUT HERE]

As can be seen, it is never profitable to promote to segments 1, 2 and 5. The result is not surprising since segments 1 and 5 do not respond to promotions and segment 2 merely takes advantage of promotions to buy more of their favorite brand at the lower price.[12]

Promoting to segments 3 and 6 can be profitable if the coefficient of promotional leverage (p for loyals, k for switchers) is greater than M_1/M_2. That is, if B_1 can preempt enough sales of other brands, the loss in profit per unit due to promotion can be offset.

Notice that this condition implies an advantage to smaller share brands. To see this, consider brand B_1 with purchase probability π_1 and brand b_2 with purchase probability π_2 such that $\pi_1 > \pi_2$. If B_1 and B_2 each have opportunities to promote alone, they can experience coefficients of prmotional leverage (k's) equal to $1/\pi_1$, and $1/\pi_2$, respectively. Since $\pi_1 > \pi_2$, we know that $1/\pi_2 > 1/\pi_1$. Hence, it is more likely that the k associated with B_2 (i.e., $1/\pi_2$) will satisfy the condition $k > M_1/M_2$ than that the k associated with B_1 (i.e., $1/\pi_1$) will.[13] This relationship holds even if other brands are promoting.

If we let $K = \sum_{\substack{B_j \in \hat{B} \\ B_j \neq B_1 \text{ or } B_2}} \pi_j$ = the sum of the probabilities of purchase associated with promoted brands, then the coefficients of promotional leverage resulting from the promotion of B_1 or B_2 are $1/(K + \pi_1)$ and $1/(K + \pi_2)$, respectively. Again, $\pi_1 > \pi_2$ implies $1/(K + \pi_2) > 1/(K + \pi_1)$.

The intuition behind this result follows from two sources. First, if a brand begins with a very high purchase probability, it simply does not have as much to gain as a brand with a very low probability of purchase. Second, we must consider the loss of profit (occasioned by dropping from M_1 to M_2) on those sales that would have been made had there been no promotion at all. For B_1, those losses amount to $\pi_1(M_1 - M_2)$. For B_2, those losses are only $\pi_2(M_1 - M_2)$. The combination of B_2's larger proportional sales gain with its smaller profit loss to regular customers provides its advantage.

The conditions for profitability of promotion to segments 4 and 7 are similar to those for segments 3 and 6. Notice that it is possible that it would be profitable to promote to segments 4 and 7 even though it was not profitable to promote to segments 3 and 6. This happens because stockpiling behavior increases the advantage to be gained from the preempted sales. Unles $M_2 < 0$ (negative profit per units sold on promotion), it is always profitable to promote to Segment 8.

For promotions to be profitable, however, it is not sufficient that there simply exist one or more segments to whom it is profitable to promote. Rather, the total mix of customers must be such that the gains from the profitable segments more than compensate for the losses to the unprofitable segments. If we take the derivative of total profit with respect to α we get:

$$\frac{2\ G_T}{\partial\alpha} = \sum_{s=1}^{8} \frac{S_s \pi_s n_s}{[1 + (n_s - 1)(\alpha + \beta_s)]^2} \quad \{ k_s [1 + \beta_s (n_s - 1) M_2 - M_{1s} \}$$

This expression provides a set of segment-specific weights

$$\frac{S_s \pi_s n_s}{[1 + (n_s - 1)(\alpha + \beta_s)]^2}$$

which can be applied to the segment specific profitability conditions

$$k_s [1 + \beta_s (n_s - 1)] M_2 - M_{1s}$$

to come up with a single condition by which to judge the profitability of promoting to the market as a whole.

In this section, we have developed conditions for promition profitability. In exploring those conditions we noted that they are more likely to hold for smaller share brands. This is in contrast to Blattberg, Eppen and Lieberman's (1981) contention (based on a retailer inventorying costs argument) that larger share brands should be more heavily promoted.

In the next section we consider the impact of promotional offers on dynamically evolving market sales. Typical sales patterns for each segment are suggested. Actual sales in two product categories are then scrutinized.

SALES PATTERNS

Each of the possible promotional responses will yield a characteristic pattern of sales. Figure 10 presents a typical sales pattern for each of the first seven segments from the point of view of maufacturer 1. Segments 1 and 5, which are not deal prone, exhibit a constant level of sales equal to P_1S_1 and π_1S_5 respectively(See Figure 10A). Recall that P_1 is the proportion of consumers loyal to B_1, π_1 is the probability that a switcher will choose B_1 and S_s is the number of consumers in segment s.

[FIGURE 10 ABOUT HERE]

Segments 3 and 6 respond to a promotion by restricting their selection to one of those brands being promoted. The typical levels for sales in these segments are P_1S_3 and π_1S_6 respectively. Sales show a sharp increase (equivalent to typical sales times the coefficient of promotional leverage) when B_1 is promoted, but drop to zero when some other brand is promoted.

The typical level of sales for segment 2, the stockpilers, is

$\frac{P_1 S_2}{1 + (n-1)\alpha}$. Recall from the model development section that only $1/(1+(n-1)\alpha)$ are expected to be vulnerable to promotion at any given point in time. (The other $\frac{(n-1)\alpha}{1+(n-1)\alpha}$ of segment 2 is using products that were stockpiled earlier.) When B_1 is promoted, we expect an n-fold increase in sales. We do not expect to see a pronounced trough in sales to this segment following a promotion. Rather, we expect an overall lowering of the typical level of sales as the dynamic process of inventory storage, depletion and repurchase evolves for each consumer.

Note that our expectation of no pronounced trough following a promotion is in direct contrast with the stated expectations of Blattberg, Eppen and Lieberman (1981) and Neslin and Shoemaker (1983). Their expectation of a pronounced trough probably arises from consideration of individual behavior. Having stockpiled a product, an individual probably will be out of the market for some time. Hence, there will be an immediate depression in that individual's purchases. However, if promotions occur frequently, the timing of individuals' stockpilings and withdrawals from the market will be staggered. The net effect across consumers will be an overall lowering of sales. Consider Figure 10. We see that, if consumers synchronized their purchase patterns, sales would evolve as in Figure 10C. Everyone would then be out of the market for the next few purchase cycles while they consumed their stockpiles. If, however, consumers respond to promotions continually and in an unsynchronized fashion, "individual troughs" will be occuring continually, leading to a sales pattern like that cross hatched in Figure 10D.

FIGURE 10: TYPICAL SALES PATTERNS BY SEGMENT

Figure 10A: SEGMENTS 1 OR 5, NOT DEAL PRONE

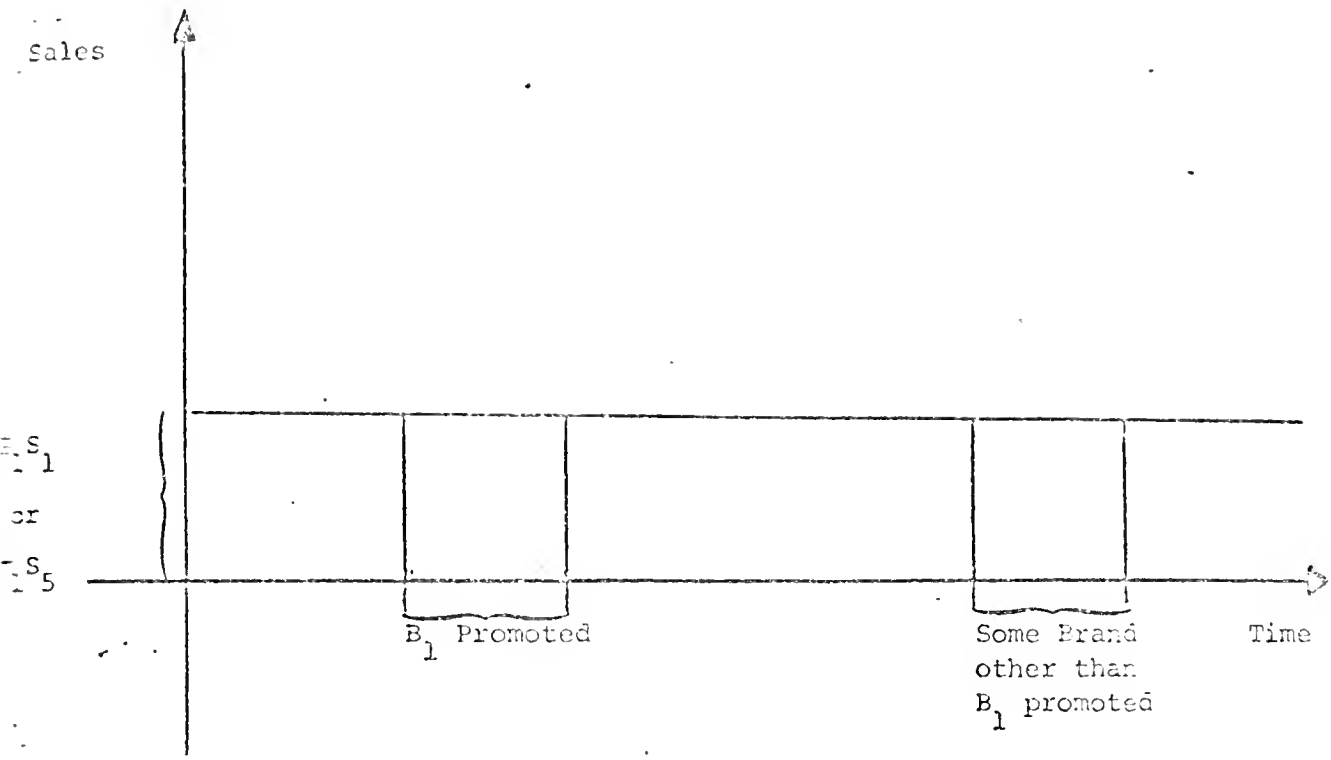


Figure 10B: SEGMENTS 3 OR 6, SWITCHING

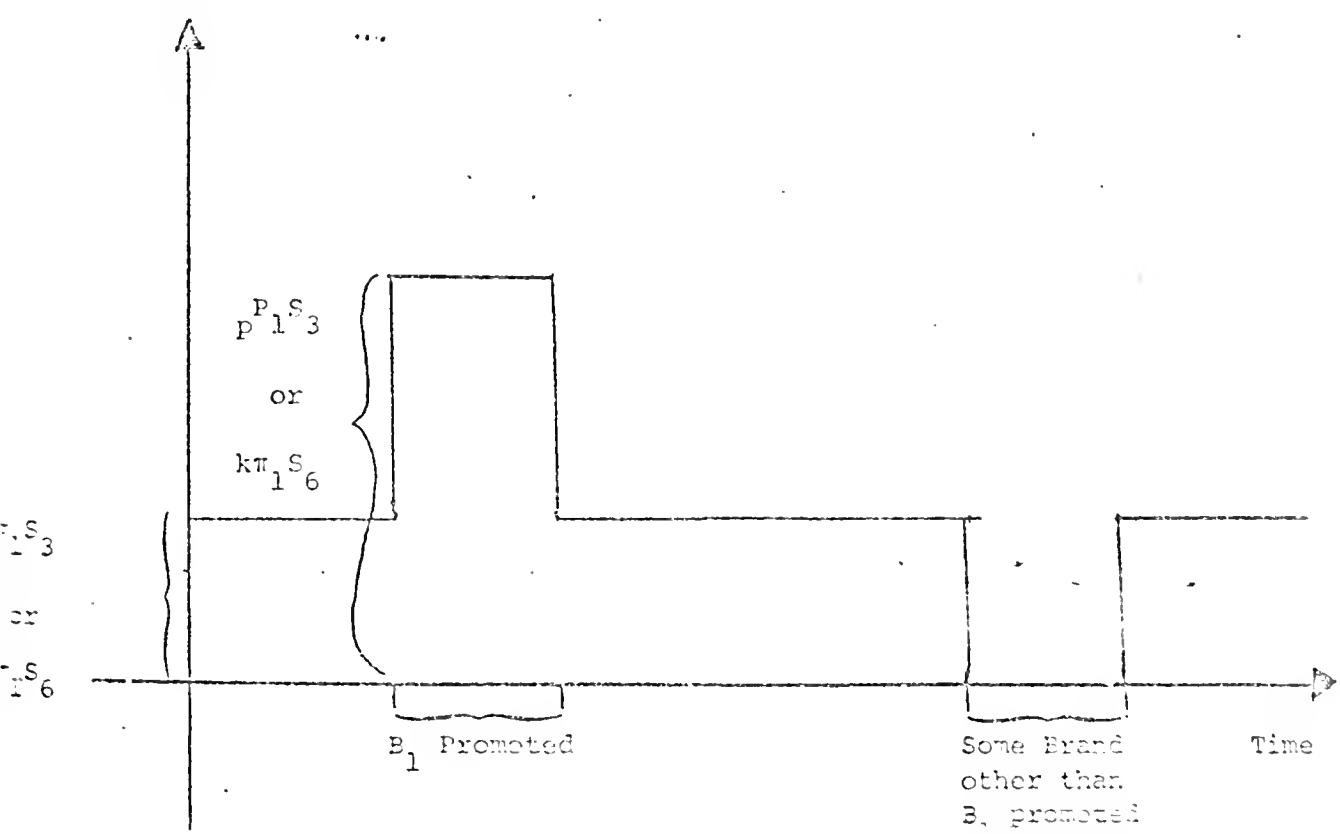


FIGURE 10: TYPICAL SALES PATTERNS BY SEGMENT

Figure 10C: SEGMENT 2, STOCKPILING WITH PRONOUNCED TROUGHS

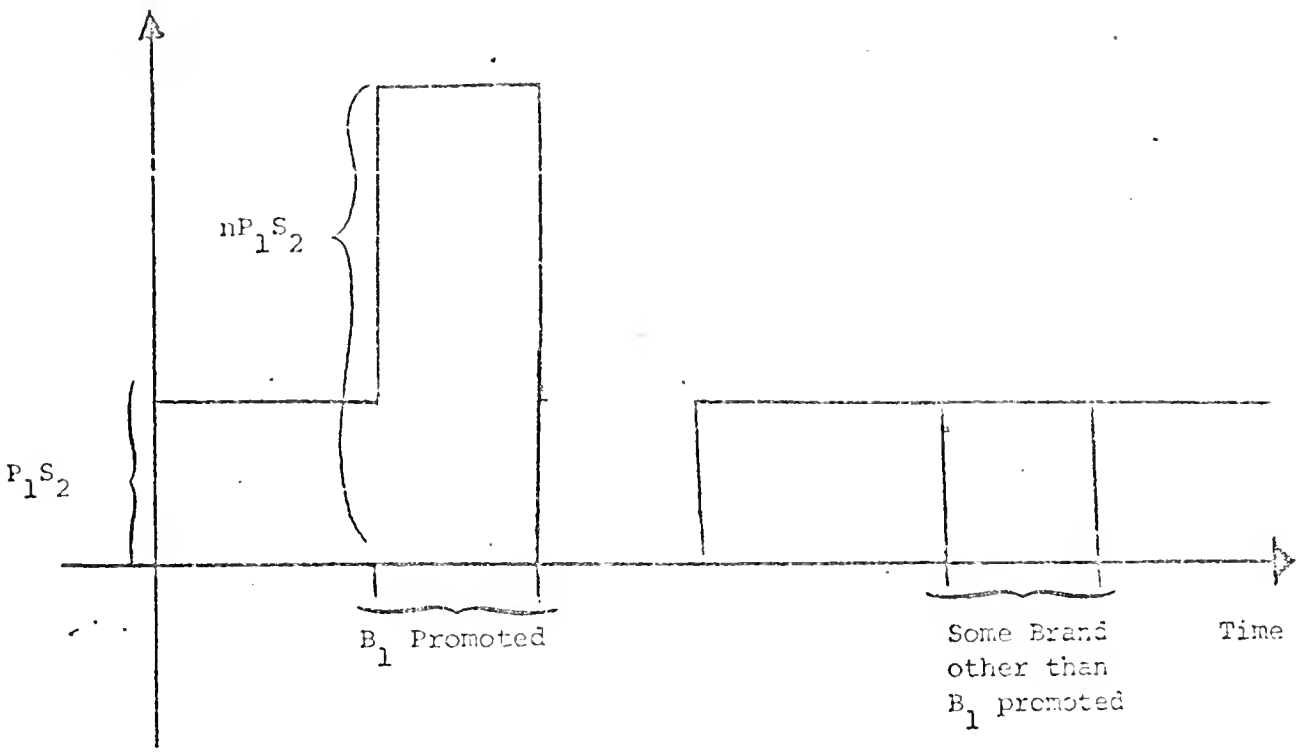


Figure 10D: SEGMENT 2, STOCKPILING WITHOUT PRONOUNCED TROUGHS
(Sales Pattern Marked with Cross Hatching)

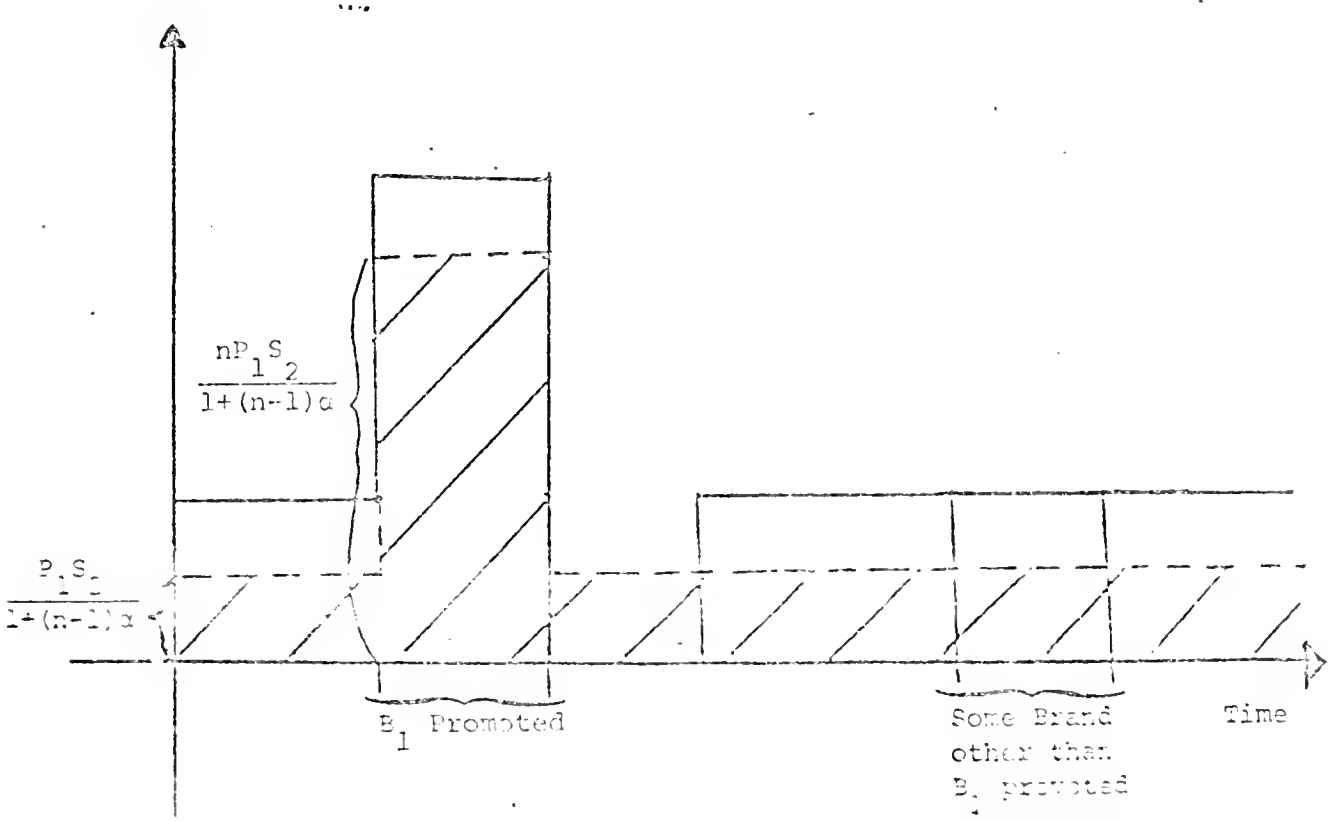
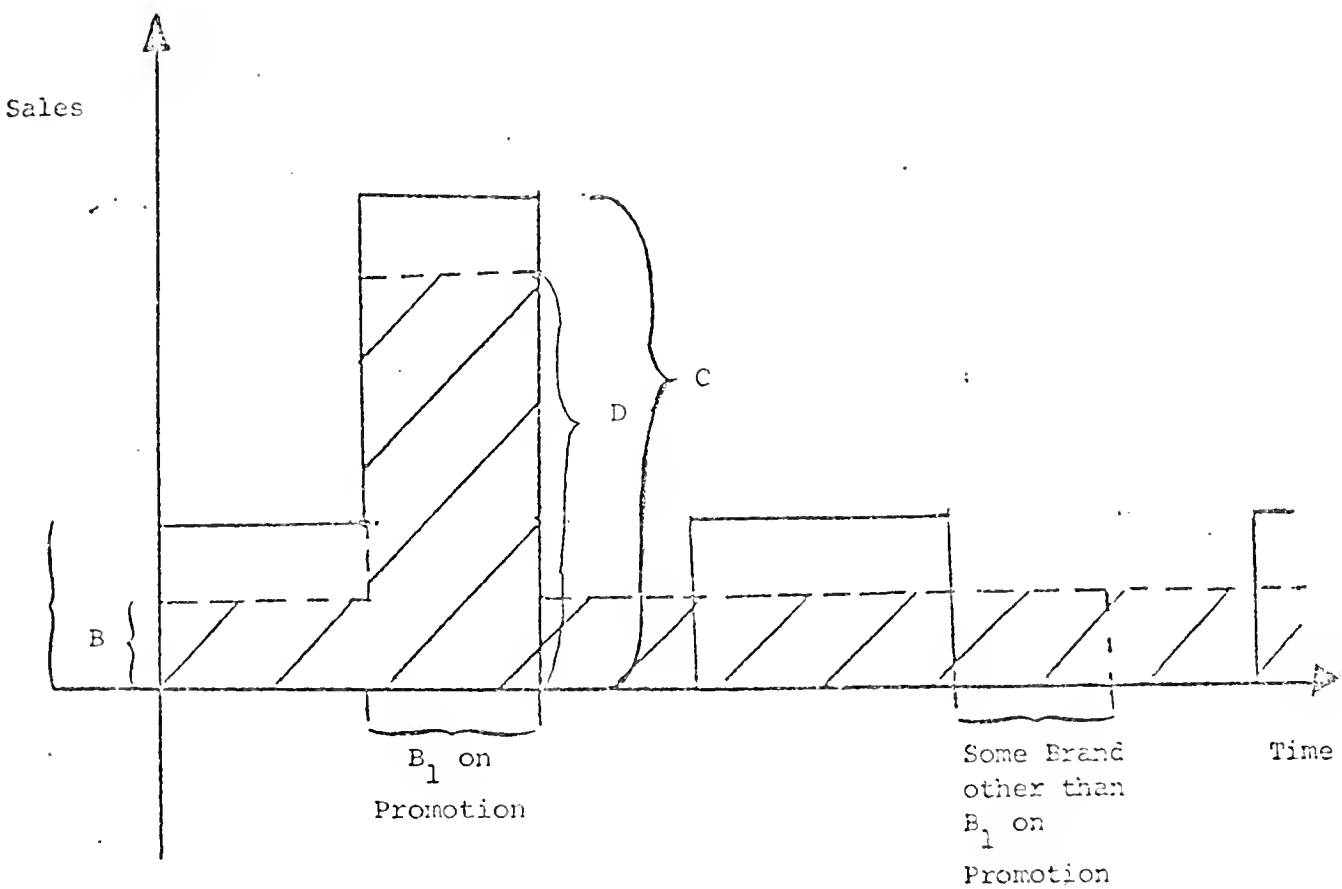


FIGURE 10: TYPICAL SALES PATTERNS BY SEGMENT

Figure 10E:
SEGMENTS 4 OR 7, SWITCHING AND STOCKPILING WITHOUT PRONOUNCED TROUGHS
(Sales Pattern Marked with Cross Hatching)



$$A = P_1 S_4 \quad \text{or} \quad \pi_1 S_7$$

$$B = \frac{P_1 S_4}{1+(n-1)(\alpha+\beta)} \quad \text{or} \quad \frac{\pi_1 S_4}{1+(n-1)(\alpha+\beta)}$$

$$C = npP_1 S_4 \quad \text{or} \quad nk\pi_1 S_7$$

$$D = \frac{npP_1 S_4}{1+(n-1)(\alpha+\beta)} \quad \text{or} \quad \frac{nk\pi_1 S_7}{1+(n-1)(\alpha+\beta)}$$

We could find no published evidence of pronounced troughs following promotional sales peaks in market level data. Even Neslin and Shoemaker (1983) who expect, a priori, to find a prounounced trough in the data they examine, are forced to amend their model because sales suffered "no appreciable drop" (Neslin and Shoemaker, 1983, p. 30) after the promotion.

This phenopmenon of a generally depressed level of sales resulting from stockpiling probably contributes to the difficulty of what Kuehn and Rohloff (1967) call "bump analysis". Such analyses establish a trend line for a brand based on its sales prior to the promotion. Departures from this trend line are then used to estimate the impact of a specific promotiom after it has occured. With the level of sales depressed by a factor of $1/(1 + (n-1)(\alpha+\beta))$, the trend line will be sensative to the amount of promotional activity $(\alpha + \beta)$ and the extent of stockpiling (n).

Segments 4 and 7, who stockpile and restrict their choice to promoted brands, exhibit all the above properties. The typical level of sales is depressed by $\frac{1}{1+(n-1)(\alpha+\beta)}$. When B_1 is promoted, sales rise by a factor of n times the coefficient of promotional leverage. When competitors promote, sales fall to zero. (See Figure 10D.)

Figures 11 and 12 represent actual sales data for two product classes. The stars along the horizontal axis represent the periods of promotion for the brand whose sales are graphed.

[FIGURE 11 ABOUT HERE]

Figure 11 represents the sales of three snack foods in a single market. Notice the small, but relatively constant band of sales. That probably represents segments 1 and 5.

The enormous peaks that accompany promotions are a little puzzling. They are probably not from segments 3, 4, 6 or 7. Sales for one brand should fall away when another is being promoted. There is no evidence of such sales depressions in this data. Since this is a snack food, it is highly unlikely that these consumers are loyal to a single brand and hence predominantly Segment 2. This product class probably has a large Segment 8. These consumers essentially do not buy in the product class (or do not buy extra units and consume them immediately) unless some brand is promoted.

Assuming that $M_2 > 0$, it is likely that promotion in this product class as a whole is profitable. The unprofitable segments 1 and 5 are probably counterbalanced by the attractive Segment 8.

[FIGURE 12 ABOUT HERE]

Figure 12 represents sales for two brands of a household cleaner in a single market. Notice that the level of sales is virtually unchanged by a brand's own promotions or by the other brand's promotions. There is no evidence of peaks or troughs in sales. This product class is probably largely composed of consumers from segments 1 and 5. It is never profitable to promote to those consumers.

In the two examples of actual sales patterns just shown, the implications for profitability are fairly clear cut. In many cases the sales patterns will provide less definitive direction for promotion management. However, as a first step in a larger analysis, an examination of the sales pattern could prove useful.

FIGURE 11: SALES PATTERNS FOR SNACK FOODS WITH PROMOTIONS IDENTIFIED

= Indicates Period of Promotion

Figure 11A: BRAND A - PROMOTIONAL ACTIVITY RESTRICTED TO JULY '79 - FEB. '80

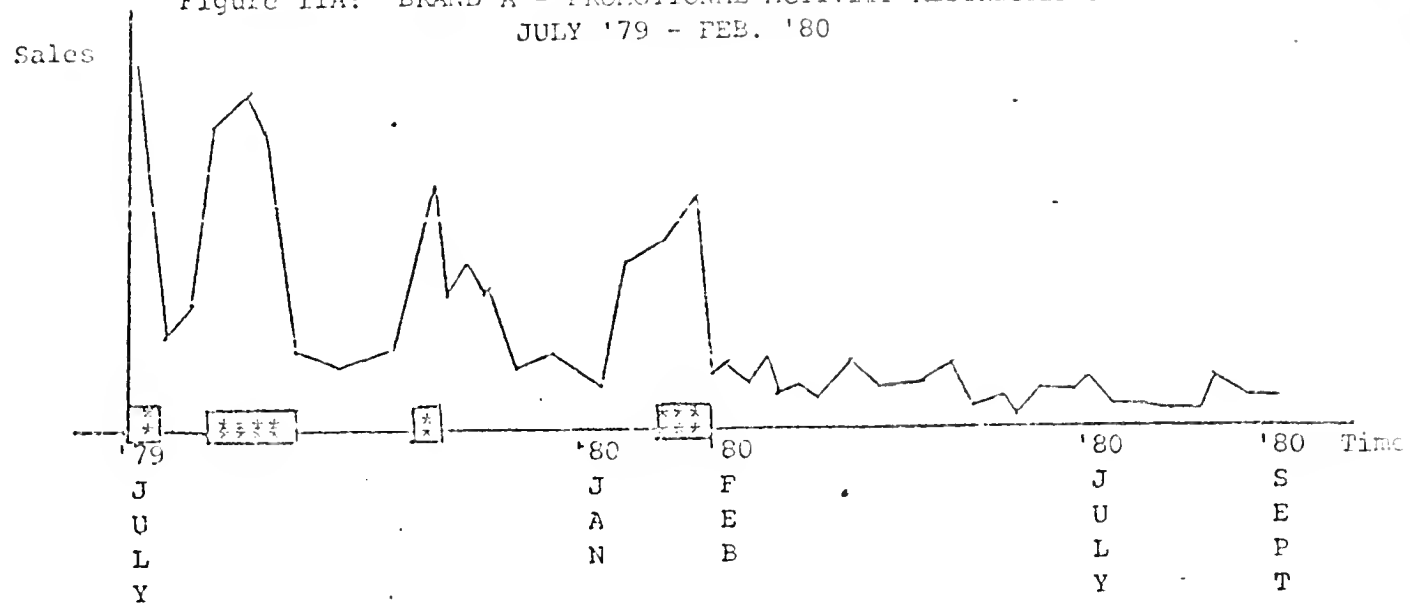


Figure 11B: BRAND B - PROMOTIONAL ACTIVITY RESTRICTED TO FEB '80 - SEPT '80

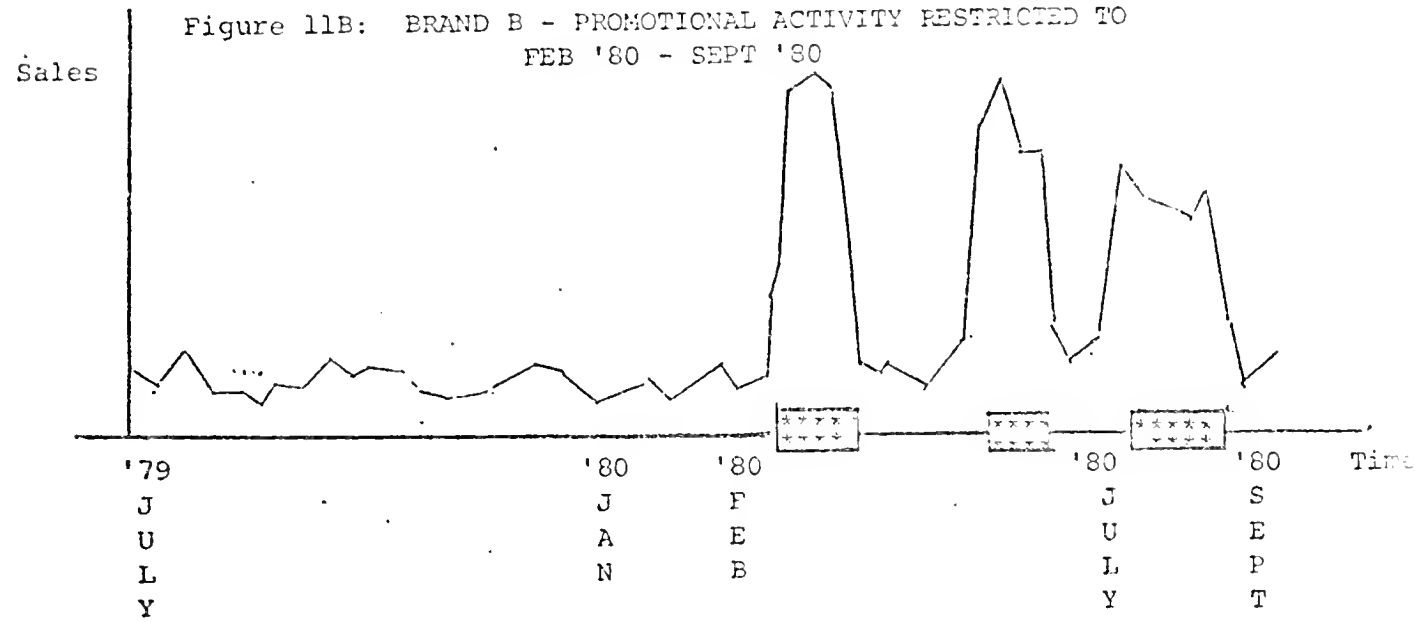


Figure 11C: BRAND C - PROMOTIONAL ACTIVITY JULY '79 - SEPT '80

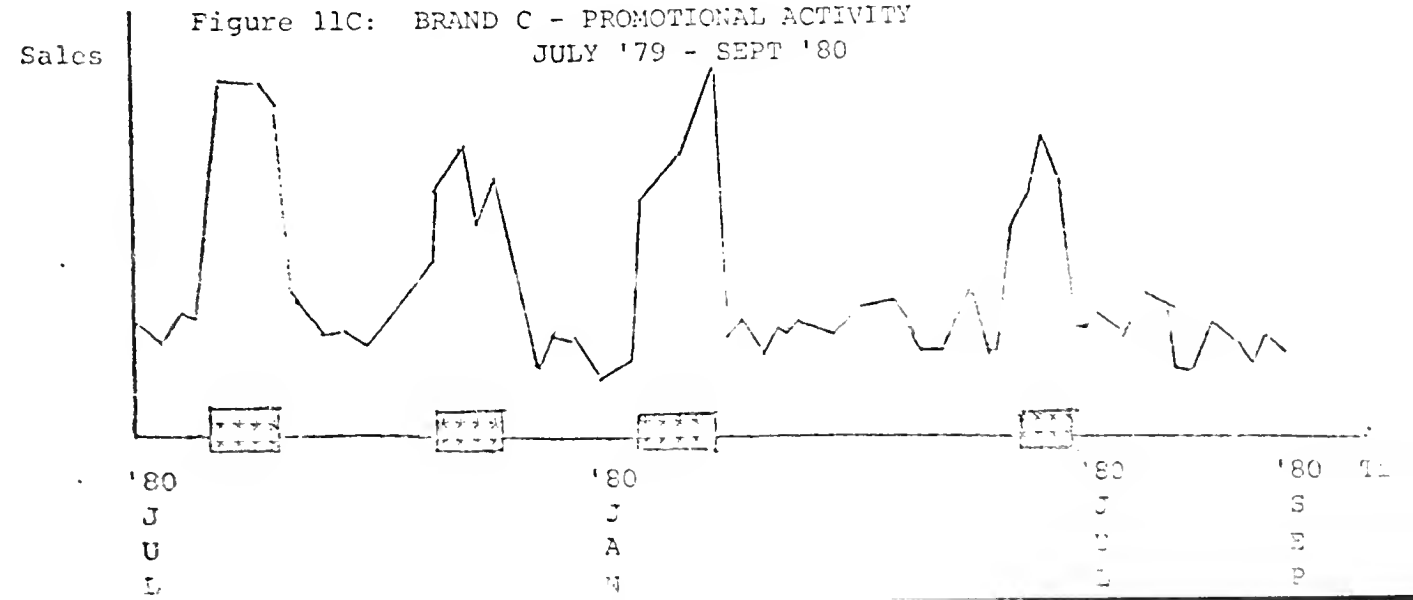
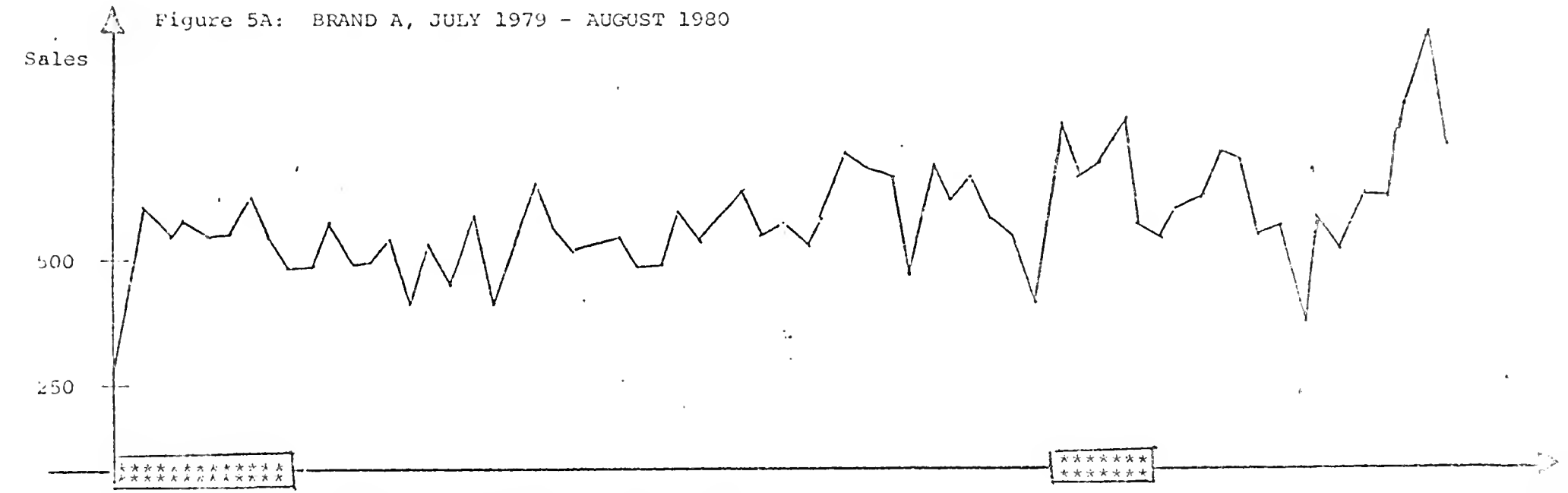
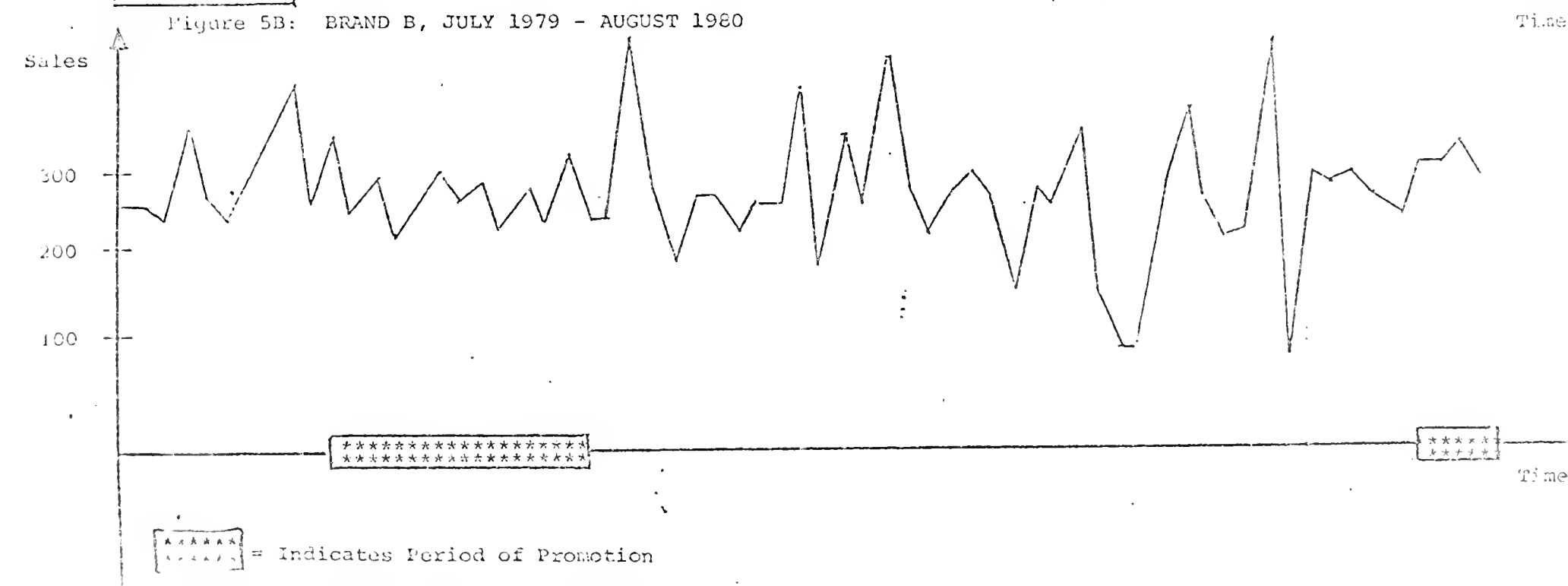


FIGURE 12: SALES PATTERNS FOR HOUSEHOLD CLEANERS WITH PROMOTIONS IDENTIFIED





SUMMARY AND CONCLUSIONS

In this paper, we incorporated both switching and stockpiling in a model of consumer response to promotions. We estimated expected profits from each of eight possible consumer segments and then aggregated to estimate total profits.

Two different consumer stockpiling policies were examined. We first assumed that, having purchased a stockpile of the product, consumers will ignore all promotions until they have consumed that stockpile. When they exhaust their stockpile they return to the market and respond to promotions as before. Alternately, we allowed consumers to restock whenever they observed a promotion. By assuming that the times between promotional offers are distributed exponentially, the consumer's stockpiling behavior becomes a renewal process. Analysis of that renewal process showed that even with relatively few competitive promoters, infrequent promotions, and low levels of stockpiling, an unexpectedly large proportion of purchases will be made on promotion.

We found that the question of whether a manufacturer should promote or not turns on whether there are a sufficient number of consumers in the marketplace who can be induced to switch brands for the promotion. Stockpiling, with no switching, was found to have a negative impact on profitability. However, if switchers can be induced to stockpile, the advantage to be gained for switching is enhanced. We also found that smaller share brands are more likely than larger share brands to find promotion profitable.

Finally, we proposed typical sales patterns for each of the consumer segments and argued that there should be no pronounced trough following a promotional sales peak. We examined actual sales data and drew inferences about the segment structure and profitability of promotions in two different product classes.

This research has attempted to synthesize and extend our current understanding of the phenomenon of consumer promotions. Many stones have been left unturned. Probably the most important direction for research is the incorporation of the retailer into the model. Only a small fraction of the promotions taken by retailers are passed along to consumers. Appropriately managing the manufacturer/retailer interaction could yield significant gains.

A second critical extension of this work would allow promotions to vary in intensity and target. As one considers the vast range of promotinal instruments (from shelf-talkers to end aisle displays, from bounce-back coupons to drastic price reductions, etc.), it is clear than an important determinant of promotion profitability has yet to be modeled.

We have also left untouched the question of cannibalization. What percentage of the predicted sales gain will come from other products in the promoting manufacturer's line? To address this question, we must develop an understanding of the particular market's structure. That understanding would also allow us to make more educated estimates of the coefficient of promotional leverage.

Finally, we need to better understand the ways in which consumers process information about promotions. Do consumers, as Rothchild and Gaddis (1981) propose, develop a tendency to repurchase a product bought on promotion? Or, on the contrary, do consumers develop a tendency to avoid a product bought on promotin as Scott (1978), and Dodson, Tybout and Sternhal (1978) propose?

We took as a given in this model the parameter a (that proportion of the time a product could be on promotion without degrading consumers' and retailers' perception of its regular price). What are the determinants of this parameter? How does promotion frequency affect consumers' and reailers' information processing?

We suggested that perhaps the loyal/switcher dichotomy in the basic segmentation scheme might be analogous to the preprocessed-decision/process-on-the-spot dichotomy proposed by Bettman and Zins (1977). If this is true, what are the implications for designing promotinal instruments to exploit that understanding? Do frequent promotions move consumers from one category to the other? Would such movement matter?

In this paper, we have taken as a given that competitors are offering promotions ß of the time. In a subsequent paper (McAlister 1983), we examine the popular hypothsis that all manufacturers might be made better off if all ceased promoting. Suprisingly, we find that this is not the case for very small share brands. Given that promotion is inevitable, we go on to consider the problem of managing promotions for more than one brand in a product line and for sequencing promotions across time.

FOOTNOTES

1 "Car-Rental Firms Sling it Out with Golf Balls and Tote Bags", Wall Street Journal, December 15, 1982, p. 29.

2 From Robert L. Simison's article "Why are Auto Sales Strictly for the Birds? Just Ask Any Pigeon", Wall Street Journal, December 15, 1982, pages 1

3 "Manufacturers Getting Snared in the 'Trade Promotions Trap'", Marketing News, March 4, 1983 and 17.

4 Marketing Communications, August, 1981.

5 "Learning" is, of course, a very important first order effect when considering the promotion of new products. In this analysis, recall we consider only the promotion of mature products.

6 Essentially we are saying that we have not included all of the relevant competitors in our definitions of the product class. For example, consider the product class canned fruit. By offering a promotion on a can of fruit we might persuade a consumer who typically purchases fresh fruit to switch to canned fruit to take advantage of the savings. One might want to restrict the body of the analysis to canned fruite purchasing behavior and lump all the fresh fruit buyers who will only buy canned fruit on promotion into this single category, Segment 8 unless:

1. Purchase within the product class was stimulated by the promotions, or
2. Extra purchases stimulated by the promotion are consumer during the purchase cycle having no negative effect on future demand.

7 Here we assume that it is not the case that all brands are being promoted at the same time

8 Loyal consumers could also be represented by a vector of preference intensities. The vector for a consumer loyal to brand B_j would have $\pi_j = 1$ and all other π's + 0. we decided to retain a titular distinction between loyals and switchers for two reasons. First, Blattberg and Sen (1976) found the distinction useful in their segmentation of consumers. Second, Bettman and Zins (1977) note two different information processing styles among consumers in a supermarket. Some consumers have preprocessed decisions. In the supermarket they merely select that brand which is their favorite. Other consumers are characterized by an acceptable set of brands. These consumers actively process information relative to those brands while in the market

One might hypothesize that those consumers we designated "Loyal" have preprocessed their decision and that our "Switchers" process on the spot. (Thanks to Kent Nakamoto for suggesting this hypothesis.)

9 Note that both Assumption 14 and Assumption 17 are manifestations of an assumption of independence of irrelevant alternatives (IIA). One could relax the IIA assumption by positing a structure of similarities among the brands and devising an appropriate expression for the multipliers p and k. Since

much of the subsequent analysis will not be dependent on the particular functional form of these mutipliers, it was not felt that IIA severely compromised this work.

10 p and k will be constant across time if preferences are stable across time and if the level of competitive promotional activity is constant across time.

11 To see this, let Θ_t be the proportion vulnerable at time t, then

$$\Theta_t = \Theta_{t-1}(1-\alpha) + (1-\Theta_{t-1})\frac{1}{n-1}$$

That is, of the Θ_{t-1} vulnerable in t-1, we would expect $(1-\alpha)$ to not observe a promotion during t-1 and hence remain vulnerable. Further, we would expect $1/(n-1)$ of those invulnerable in period t-1 to have exhausted their inventories and become vulnerable in period t. If we solve the above equation for an equilibrium value of Θ, we get

$$\Theta = \Theta(1-\alpha) + (1-\Theta)\frac{1}{n-1}$$

or

$$\Theta = \frac{1}{1+(n-1)\alpha} .$$

12 Note that it is behavior such as this that Blattberg, Eppen and Lieberman (1981) propose as the motivation for retailers to promote to consumers. Their inventory holding cost argument makes sense for the retailer/consumer interface. However, we are concerned with the relationship between the manufacturer and the consumer in this analysis.

13 Here we assume that M_1 and M_2 are constant across brands.

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